Physics Letters B 747 (2015) 27–35

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

A new spin on neutrino quantum kinetics

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ARTICLE INFO

Article history:
Received 16 March 2015
Received in revised form 25 April 2015
Accepted 29 April 2015
Available online 14 May 2015
Editor: W. Haxton

ABSTRACT

Recent studies have demonstrated that in anisotropic environments a coherent spin-flip term arises in the Quantum Kinetic Equations (QKEs) which govern the evolution of neutrino flavor and spin in hot and dense media. This term can mediate neutrino–antineutrino transformation for Majorana neutrinos and active-sterile transformation for Dirac neutrinos. We discuss the physical origin of the coherent spin-flip term and provide explicit expressions for the QKEs in a two-flavor model with spherical geometry. In this context, we demonstrate that coherent neutrino spin transformation depends on the absolute neutrino mass and Majorana phases.

1. Introduction

The evolution of an ensemble of neutrinos in hot and dense media is described by an appropriate set of quantum kinetic equations (QKEs), accounting for kinetic, flavor, and the often neglected spin degrees of freedom [1–10]. QKEs are the essential tool to obtain a complete description of neutrino transport in the early universe, core collapse supernovae, and compact object mergers, valid before, during, and after the neutrino decoupling epoch (region). A self-consistent treatment of neutrino transport is highly relevant because in such environments neutrinos carry a significant fraction of the energy and entropy, and through their flavor- and energy-dependent weak interactions play a key role in setting the neutron-to-proton ratio [11], a critical input for the nucleosynthesis process.

Recent studies [8,10] have demonstrated that the QKEs acquire a coherent spin-flip in regions where the spatial (anti)neutrino fluxes are anisotropic or where there exist anisotropic matter currents. Such anisotropy can exist in a core-collapse supernovae or compact object merger environments. This spin-flip term can mediate neutrino–antineutrino transformation for Majorana neutrinos and active-sterile transformation for Dirac neutrinos. Moreover, it was shown in Ref. [7] that a general treatment of neutrino ensembles should include correlations that pair neutrinos and antineutrinos of opposite momenta. The coupling to these new densities to the standard density matrices has been worked out explicitly in Ref. [10]. In this work we neglect these terms as their effect primarily generates coherence of opposite-momentum neutrinos only for very long-wavelength modes, with $\lambda_{\text{de Broglie}} \sim \lambda_{\text{scale-height}}$, where $\lambda_{\text{scale-height}}$ is the length scale characterizing a given astrophysical environment. Significant feedback effects from the long-wavelength modes could alter the analysis presented below, and this deserves a separate study.

In this letter we further elaborate on the terms of the QKEs describing coherent neutrino evolution (i.e. neglecting inelastic collisions). The novel aspects of this work are:

- We discuss the physical origin of the coherent spin-flip term in the framework of an MSW-like effective Hamiltonian, in analogy to the spin-(flavor) oscillations induced by neutrino magnetic moments in a magnetic field.
- We provide explicit expressions for the coherent QKEs in a two-flavor model with spherical geometry, amenable for a computational implementation. This is the first step towards a realistic exploration of the impact of helicity oscillations in astrophysics environments.

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http://dx.doi.org/10.1016/j.physletb.2015.04.066

- We point out the dependence of the QKEs (through the neutrino–antineutrino conversion term) on the neutrino absolute mass scale and Majorana phases. We also compare and contrast neutrino-less double beta decay and neutrino spin transformation in astrophysical environments as probes of these parameters.

2. Spin-mixing term

Refs. [8,10] have pointed out that in anisotropic environments the QKEs entail a new term that drives coherent conversion between different helicity states (of any flavor). An important feature of the new term is that it induces qualitatively different effects for Dirac and Majorana neutrinos. In the Dirac case, the mixing term converts active left-handed neutrinos to sterile right-handed states. On the other hand, in the Majorana case the mixing term enables conversion of neutrinos into antineutrinos. Given the potentially high impact of the spin-flip term, here we discuss its physical origin in a framework that does not involve the intricacies of non-equilibrium quantum field theory. Indeed, as argued below, the basic physics of this term can be understood in the case of one-flavor Dirac neutrinos even at the first-quantized level.

Physically, spin oscillations are induced by the axial-vector potential generated by forward scattering of neutrinos on the background matter and background neutrinos themselves. To illustrate this point, let us first consider the evolution of neutrinos in *external* chiral four-vector potentials $\Sigma^{\mu}_{L,R}$ (we will give their explicit expressions later on). Since our discussion parallels the analysis of spin-flip transition induced by a neutrino magnetic moment in an external magnetic field [12,13], we also include in the interaction Lagrangian the familiar magnetic-moment term. Suppressing flavor indices (μ_ν and $\Sigma_{L,R}$ are matrices in flavor space) the interaction Lagrangian is given by

$$\mathcal{L}_{\rm int} = -\bar{\nu}_L \not{\Sigma}_R \nu_L - \bar{\nu}_R \not{\Sigma}_L \nu_R + \left(\frac{\mu_\nu}{2}\, \bar{\nu}_R \sigma_{\mu\nu} F^{\mu\nu} \nu_L + \text{h.c.} \right) . \tag{1}$$

The Majorana case is obtained by replacing $\nu_R \to \nu_L^c$, $\Sigma_L \to -\Sigma_R^T$, and setting to zero the diagonal elements μ_ν^{ii} (for Majorana neutrinos $\mu_\nu^{ji} = -\mu_\nu^{ij}$). Given this interaction, our goal is to obtain an effective Hamiltonian in spin(-flavor) space, with off-diagonal components giving the helicity mixing [13]. Since the essential physics of spin oscillations is already present in the one-flavor case, to keep the discussion as simple as possible we consider the case of one-flavor Dirac neutrinos, with real magnetic moment. In this case the interaction Lagrangian is

$$\mathcal{L}_{\rm int} = \frac{\mu_\nu}{2}\, \bar{\nu} \sigma_{\mu\nu} F^{\mu\nu} \nu - \frac{1}{2} \bar{\nu} \not{\Sigma}_V \nu - \frac{1}{2} \bar{\nu} \not{\Sigma}_A \gamma_5 \nu \ , \tag{2}$$

where we have defined the vector and axial-vector potentials as $\Sigma^{\mu}_{V,A} \equiv \Sigma^{\mu}_L \pm \Sigma^{\mu}_R = (\Sigma^0_{V,A}, \vec{\Sigma}_{V,A})$.

In a first-quantized approach [12], the Dirac Hamiltonian corresponding to the interaction (2) is

$$H = H_0 + \Delta H \ , \qquad H_0 = \hat{p} \cdot \vec{\alpha} + \beta m \ , \qquad \Delta H = \mu_\nu\, \beta\, \vec{\Sigma} \cdot \vec{B} + \left(\Sigma^0_V - \vec{\Sigma}_V \cdot \vec{\alpha} \right) + \left(\gamma_5 \Sigma^0_A - \vec{\Sigma}_A \cdot \vec{\Sigma} \right) \ , \tag{3}$$

with $\beta = \gamma^0$, $\vec{\alpha} = \gamma^0 \vec{\gamma}$, and $\vec{\Sigma} = \text{diag}(\vec{\sigma}, \vec{\sigma})$. Defining the helicity operator $h \equiv \hat{p} \cdot \Sigma$, already at this level one sees that while $[H_0, h] = 0$, in general $[\Delta H, h] \neq 0$, unless $\vec{\Sigma}_A$ and $\vec{B}$ are parallel to the momentum $\vec{p}$. So the energy eigenstates are in general mixtures of helicity eigenstates, and we reach the conclusion that magnetic fields *and/or* axial-vector potentials transverse to the direction of motion induce helicity oscillations.

To quantify the helicity mixing effect, it is more convenient to work within the second-quantized quantum field theory approach [13]. One can define the 2×2 effective Hamiltonian in helicity space $\mathcal{H}_{hh'}$ by computing transition amplitudes between massive neutrino states labeled by momentum $\vec{p}$ and helicity $h \in \{L, R\}$ namely

$$\langle \vec{p}\,', h' \,|\, \vec{p}, h \rangle \equiv -i (2\pi)^4\, 2E_{\vec{p}}\, \delta^{(4)}(p - p')\, \mathcal{H}_{h'h}(p). \tag{4}$$

To first order in the interaction (2) and to all orders in $m/|\vec{p}|$ (with the notation $p \equiv |\vec{p}|$, $E = \sqrt{m^2 + p^2}$), following the steps outlined in Appendix A we find

$$\mathcal{H}_{LL}(p) = \frac{E+p}{4E} \Big\{ -4r(p)\, \mu_\nu\, \hat{p} \cdot \vec{B} - (1 - r(p)^2) \Sigma^0_A + (1 + r(p)^2) \hat{p} \cdot \vec{\Sigma}_A + (1 + r(p)^2) \Sigma^0_V - (1 - r(p)^2) \hat{p} \cdot \vec{\Sigma}_V \Big\} \tag{5}$$

$$\mathcal{H}_{RR}(p) = \frac{E+p}{4E} \Big\{ +4r(p)\, \mu_\nu\, \hat{p} \cdot \vec{B} + (1 - r(p)^2) \Sigma^0_A - (1 + r(p)^2) \hat{p} \cdot \vec{\Sigma}_A + (1 + r(p)^2) \Sigma^0_V - (1 - r(p)^2) \hat{p} \cdot \vec{\Sigma}_V \Big\} \tag{6}$$

$$\mathcal{H}_{LR}(p) = \frac{E+p}{2E} \Big\{ (1 + r(p)^2)\, \mu_\nu\, \hat{x}_+ \cdot \vec{B} \; - \; r(p)\, \hat{x}_+ \cdot \vec{\Sigma}_A \Big\} \tag{7}$$

$$\mathcal{H}_{RL}(p) = \frac{E+p}{2E} \Big\{ (1 + r(p)^2)\, \mu_\nu\, \hat{x}^*_+ \cdot \vec{B} \; - \; r(p)\, \hat{x}^*_+ \cdot \vec{\Sigma}_A \Big\} \ , \tag{8}$$

where

$$r(p) = \frac{m}{E+p} \qquad 1 + r(p)^2 = \frac{2E}{E+p} \qquad 1 - r(p)^2 = \frac{2p}{E+p} \ , \tag{9}$$

and $\hat{x}_+ \equiv e^{i\phi_p} (\hat{x}_1 + i\hat{x}_2)$ with $\hat{x}_{1,2}$ defined so that $(\hat{x}_1, \hat{x}_2, \hat{p})$ form a right-handed triad. The choice of $\hat{x}_{1,2}$ orthogonal to $\hat{p}$ is arbitrary up to a rotation along the $\hat{p}$ axis. We use here the “standard gauge” specified by choosing the same azimuthal angle for $\hat{x}_1$ and $\hat{p}$ ($\phi_{x_1} = \phi_p$),[1] with unit vectors Cartesian coordinates expressed in terms of the polar and azimuthal angles (θ_p, ϕ_p) by

[1] In Ref. [8] a different “gauge” was used, in which a rotation by $-\phi_p$ was made in the $\hat{x}_1$–$\hat{x}_2$ plane. With this choice the phase factors $e^{\pm i\phi_p}$ disappear from all formal expressions, but the algebra to obtain dot products of $\hat{x}_{1,2}(p)$ with other vectors is more cumbersome.