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Effective field theories on solitons of generic shapes ☆

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ABSTRACT

A class of effective field theories for moduli or collective coordinates on solitons of generic shapes is constructed. As an illustration, we consider effective field theories living on solitons in the O(4) non-linear sigma model with higher-derivative terms.

1. Introduction

Effective field theory is one of the most useful tools available to date. Even the standard model, although renormalizable in its present formulation, may also be just an effective theory of Nature where possible supersymmetric and/or grand unified extensions have been integrated out. For particles of accessible energies, we can neglect gravity and consider particles on flat space as an (extremely) good approximation. This is just a consequence of the separation of scales between the particle mass and energy versus the scale of gravity, i.e. the Planck mass. Light fields do not only exist in all of spacetime but are sometimes confined to certain subspaces. For solitons hosting moduli, there is again a situation where separation of scales can be exploited; namely the mass of the soliton versus massless or light moduli. Effective field theories for moduli have been constructed for many kinds of solitons, but very often only in cases where the soliton has a simple, flat or straight shape. As examples, the effective actions for monopole moduli [1], domain-wall moduli [2–5] and for orientational moduli of non-Abelian strings [6,4,5,7] have been constructed. When solitons are particle-like such as monopoles this can describe the low-energy dynamics of the solitons in a compact way as geodesics of moduli spaces [1], while for solitons being extended objects such as domain walls or vortices, this describes field theories on their world-volume, as in the case of D-branes in string theory or more general branes. Solitons can, however, generically possess much more complicated shapes.

In this Letter we construct a first attempt of effective field theories in principle applicable to solitons of generic shapes and apply it to a class of models possessing soliton solutions of flat, spherical, cylindrical and toroidal shapes.

2. General considerations

Here we will consider a generalized framework where we expand a set of fields in eigenmodes as [2]

$$\Phi^a = \sum_n \mathcal{M}_n(\mathbf{e}_\alpha)\zeta_n^a(\mathbf{e}_i), \tag{1}$$

where ζ_n are eigenfunctions, $\mathcal{M}_n$ are moduli fields, while $\mathbf{e}_\alpha$ and $\mathbf{e}_i$ are sets of vectors in transverse (world-volume) dimensions ($\alpha = 0, 1, \ldots t$) and codimensions ($i = t+1, \ldots t+c$), respectively, of a soliton of a generic shape; see Fig. 1. For simplicity we consider only flat space in this Letter and we have made a decomposition of directions (locally) as $\mathbb{R}^{d,1} = \mathbb{R}^{t,1} \times \mathbb{R}^c$, where the $d = c + t$ spatial dimensions are split into c codimensions and t transverse dimensions.

The kinetic term in the underlying theory will give rise to a kinetic term for the moduli as

$$\int_{\mathbf{e}_i} |\nabla_\mu \Phi|^2 \supset |\nabla_{\mathbf{e}_\alpha}\mathcal{M}_n|^2 \int_{\mathbf{e}_i} |\zeta_n|^2 \propto \frac{1}{M^c}|\nabla_{\mathbf{e}_\alpha}\mathcal{M}_n|^2, \tag{2}$$

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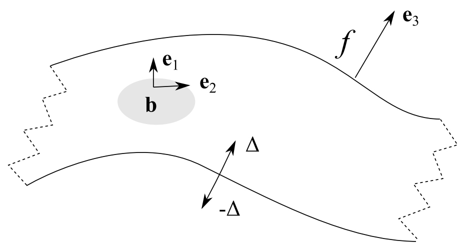

Fig. 1. Sketch of a generic soliton profile in direction $\mathbf{e}_3$. The moduli $\mathbf{b}$ live on the manifold spanned by the host soliton. The integration over the codimension is done only over a finite range $[-\Delta, \Delta]$, allowing for a generic shape of the host soliton.

where M is a characteristic mass of the soliton system and μ are all spacetime indices. For higher-order derivative terms, one similarly obtains e.g.,

$$\int_{\mathbf{e}_i} |\nabla_\mu \Phi|^2 |\nabla_\nu \Phi|^2 \supset |\nabla_{\mathbf{e}_\alpha} \mathcal{M}_n|^2 \int_{\mathbf{e}_i} |\nabla_{\mathbf{e}_i} \zeta_m|^2 |\zeta_n|^2 \propto \frac{1}{M^{c-2}} |\nabla_{\mathbf{e}_\alpha} \mathcal{M}_n|^2. \quad (3)$$

Notice the relative enhancement of this term compared to that of (2). The higher-order term induces an enhanced kinetic term in the low-energy effective theory living on the soliton.

However, the lower-order term also induces other terms in the low-energy effective theory, which will be of higher-order. These induced terms are of a different kind as they are higher-order corrections coming from integrating out massive modes propagating on the soliton. Let us consider the kinetic term, which would induce something like

$$\frac{1}{M^{c+2}} |\nabla_\alpha \mathcal{M}_n|^2 |\nabla_{\alpha'} \mathcal{M}_{n'}|^2. \quad (4)$$

This higher-order correction in the effective theory is naturally suppressed by (2 powers of) the soliton scale. Whether this term will be comparable to the higher-order terms in the theory before we take the low-energy limit on the soliton depends on the theory and the parameters.

In this Letter, we consider the higher-order terms to be numerically significant and work in the limit of very high soliton mass, where we safely can neglect the higher-order corrections coming from lower-order terms.[1]

Let us comment on integrating out the host soliton. We assume that the soliton is extended in the directions spanned by $\{\mathbf{e}_i\}$ which is taken to be orthogonal to $\{\mathbf{e}_\alpha\}$. However, integrating over all the subspace spanned by $\{\mathbf{e}_i\}$ may be problematic; but for physical reasons we need only integrate over the major energy peak of the soliton solution (say in the range $[-\Delta, \Delta]$) on which the moduli live and thus neglect the long tales that the soliton may possess; see Fig. 1. We do this for physically capturing the low-energy effective theory on the soliton and in a way that we can still use the decomposition of the transverse and world-volume coordinates locally.

Finally, we need to assess the quality of the approximation we are making, since we are taking into account corrections proportional to powers of the soliton mass coming from higher-order terms. The approximation we are making is a separation of scales between the mass of the host soliton and the energies of the moduli in the effective action living on the world-volume. The higher-order terms, if they have non-negligible coefficients, induce lower-order terms in the low-energy effective theory on the soliton which are enhanced by a factor of $(M/m)^{\delta d}$ (where δd is the difference in dimension between the higher-order term and the lower-order term while m is the typical scale of the moduli). On the other hand, as mentioned above, the lower-order terms also induce higher-order correction terms which come about from integrating out massive modes propagating along the soliton. These terms are, however, suppressed by a factor of $(m/M)^2$ (or higher). It has also been assumed all along that the derivatives in the low-energy effective theory are not too large. As long as the ratio m/M is sufficiently small, we can use just the leading-order low-energy effective theory.

Higher-order corrections coming from the lower-order terms, as mentioned above, can however be calculated systematically [7], but we will not consider them in this Letter; i.e., here we present only the leading-order effective action.

[1] Needless to say, this may not always be the case, but it is a limit we work in here for simplicity.

3. Non-linear sigma model

To illustrate our framework more explicitly, we will now specialize the considerations presented above to an O(4)-sigma model with higher-derivative terms in $3+1$ flat dimensions, which has scalar fields, n^a, of an O(4) vector, with $a = 1, \ldots, 4$ and Lagrangian density

$$\mathcal{L} = -m^4 V + c_2 m^2 \mathcal{L}_2 + c_4 \mathcal{L}_4 + \frac{c_6}{m^2} \mathcal{L}_6 + \cdots \quad (5)$$

where $\mathcal{L}_n$ is the Lagrangian density containing the n-th order derivative terms

$$-\mathcal{L}_2 = \frac{1}{2} \partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n}, \quad (6)$$

$$-\mathcal{L}_4 = \frac{1}{4} \left(\partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n}\right)^2 - \frac{1}{4} \left(\partial_\mu \mathbf{n} \cdot \partial_\nu \mathbf{n}\right)^2, \quad (7)$$

$$\mathcal{L}_6 = \mathcal{B}_\mu \mathcal{B}^\mu, \quad \mathcal{B}^\mu = \frac{1}{6} \epsilon^{\mu\nu\rho\sigma} \epsilon^{abcd} \partial_\nu n^a \partial_\rho n^b \partial_\sigma n^c n^d, \quad (8)$$

and $\mathcal{B}^\mu$ is the baryon current. Finally, an appropriate potential should be chosen for the soliton under study. There still remains a choice to be made, i.e. the codimension of the soliton under consideration. Since we consider $\mathbb{R}^{3,1}$ here, there are only two non-trivial cases: a codimension-one soliton like a domain wall or a codimension-two soliton like a vortex. We will study each in turn in the following.

3.1. Codimension-one case

We will now consider the soliton of the type which is described by a codimension-one field $\zeta(\mathbf{e}_3)$ and two moduli $\mathcal{M}_{1,2}(\mathbf{e}_1, \mathbf{e}_2)$, where the *condensate* field is a function of the direction spanned by the vector $\mathbf{e}_3$ only and the moduli are functions of two orthogonal directions $\mathbf{e}_1$ and $\mathbf{e}_2$. For concreteness we will parametrize the non-linear sigma-model field, $\mathbf{n}$, as

$$\mathbf{n} = \{\mathbf{b} \sin f, \cos f\}, \quad (9)$$

where $\mathbf{b}$ are scalar fields of a unit 3-vector ($\mathbf{b} \cdot \mathbf{b} = 1$) describing two moduli and is a function only of the orthogonal directions to the field f, i.e. $\mathbf{b}(\mathbf{e}_1, \mathbf{e}_2)$. The domain solution also possesses a position modulus, which we will not take into account in this Letter. Taking the Lagrangian densities (6)–(8) one-by-one, choosing the potential

$$m^2 V = -\frac{1}{2} m_3^2 n_3^2 + \frac{1}{2} M^2 (1 - n_4^2), \quad (10)$$

and integrating over the codimension spanned by $\mathbf{e}_3$, we get