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On effective spacetime dimension in the Hořava–Lifshitz gravity

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ABSTRACT

In this manuscript we explicitly compute the effective dimension of spacetime in some backgrounds of Hořava–Lifshitz (H–L) gravity. For all the cases considered, the results are compatible with a dimensional reduction of the spacetime to $d+1=2$, at high energies (ultraviolet limit), which is confirmed by other quantum gravity approaches, as well as to $d+1=4$, at low energies (infrared limit). This is obtained by computing the free energy of massless scalar and gauge fields. We find that the only effect of the background is to change the proportionality constant between the internal energy and temperature. Firstly, we consider both the non-perturbative and perturbative models involving the matter action, without gravitational sources but with manifest time and space symmetry breaking, in order to calculate modifications in the Stephan–Boltzmann law. When gravity is taken into account, we assume a scenario in which there is a spherical source with mass M and radius R in thermal equilibrium with radiation, and consider the static and spherically symmetric solution of the H–L theory found by Kehagias–Sfetsos (K–S), in the weak and strong field approximations. As byproducts, for the weak field regime, we used the current uncertainty of the solar radiance measurements to establish a constraint on the ω free parameter of the K–S solution. We also calculate the corrections, due to gravity, to the recently predicted attractive force that black bodies exert on nearby neutral atoms and molecules.

1. Introduction

The Hořava–Lifshitz (H–L) gravity is an alternative proposal to quantum gravity theory, recently presented in the literature, which is power-counting renormalizable [1–3]. The cost to ensure this renormalizability is to ignore the local Lorentz symmetry incorporated in Einstein's theory of gravitation and to consider different kinds of spatial and temporal scaling at very short distances, *i.e.*, in the ultraviolet (UV) regime, with this symmetry accidentally emerging in the opposite situation, namely, at the infrared (IR) regime [4]. According to this novel approach, at the UV regime an anisotropic scaling occurs, in such a way that time transforms as $t \to b^z t$ and space as $x^i \to bx^i$, where z is a dynamical critical exponent that goes to unity at large distances, which means that the validity of General Relativity is restored, at the IR scale.

The possible values of the critical exponent z establish a connection between the H–L theory and the number of dimensions that the Universe must possess at all scales. In fact, Hořava [3] calculates the dimension seen by a diffusion process, the spectral dimension d_s, as being equal to 2 at the UV scale, which is compatible with other approaches of quantum gravity [5–7]. At the opposite scale, which means in the IR regime, the spectral dimension is $d_s = 4$ [3]. The spectral dimension as a continuous function of time which links these two amounts, in the context of the H–L theory, was obtained in [8]. A general discussion about the dependence of the dimensionality with the scale ("evolving" or "vanishing" dimensions), in the context of several theories, can be found in [9]. Hořava still presents [3] heuristic scaling arguments based on the free energy of a system consisting of free massless fields to show that the effective number of topological dimensions is again equal to 2, since that the free energy found is proportional to the square of temperature.

Studies concerning H–L gravity were accomplished in several contexts, as in the analysis of the Casimir effect [11,12] and in the study of spacetime stability [13], as well as in some astrophysical

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and cosmological models [14–16], including those ones which predict a multiverse [17]. It was shown that H–L gravity is weaker than General Relativity even at high energies, generating a bouncing cosmology and an accelerating universe, naturally incorporating dark energy [18,19]. From the point of view of Astrophysics, some static and spherically symmetric black hole-type solutions of H–L gravity were found, and the simplest one was proposed by Kehagias and Sfetsos (K–S) [20], whose IR limit gives us the Schwarzschild solution. Other exact spherically symmetric solutions for the H–L gravity can be found in [21–24].

The connection between the phenomenology involving gravity and thermodynamics was studied in the early seventies of past century [26,27], and reexamined more recently [28,29]. In this context, black holes seem to be the objects of the universe in which these connections become stronger, and any modifications in the laws derived from thermodynamics due to gravitational effects must necessarily consider these structures. In this scenario, black body radiation is emblematic in view of the possible role played by that phenomenon in the construction of a quantum theory of gravity.

In this paper, we calculate explicitly the number of effective topological dimensions of the Universe by analyzing modifications in the laws of the black body radiation due to the H–L theory, at both UV and IR scales with and without gravitation sources. It is supposed that the thermal radiation comes from the dynamics of a free massless scalar field. We then calculate the Helmholtz free-energy associated with the field for the cases under consideration. Firstly, the non-perturbative model in which, due to the spacetime anisotropy required by H–L theory, the spatial derivatives of the scalar field are implemented in the action with higher orders than the temporal one.

In what follows, we will take into account the perturbative model where, besides these derivatives, we add one more term which depends on the first order spatial derivative. We use the models presented in [11] to obtain the dependence of the Helmholtz free-energy with the temperature and to determine the effective number of spacetime dimensions. Next we consider a spherical gravitational source, by considering the weak and strong field approximations of the K–S solution, in both IR and UV regimes. Finally, for the weak field regime, we will obtain constraints on the K–S ω parameter from the uncertainty in the measurements of the solar radiance. In view of the both General Relativity and H–L theories we will also calculate corrections to the thermal shift in the energy of the fundamental state of the hydrogen atom, known as dynamical Stark effect, obtaining the anomalous force of attraction upon it due to the black body radiation.

The paper is organized as follows. In Section 2, we determine modifications in the laws of the black body radiation in the context of the H–L theory without considering the presence of a gravitational field. In Section 3, we do similar calculations to the previous section, but now taking into account the presence of a gravitational field. In Section 4 we obtain the force of attraction corrected by effects due to classical and H–L gravity. Finally, in Section 5 we present the concluding remarks.

2. Black body radiation without gravitational sources

2.1. Non-perturbative model

In the H–L theory, the violation of the Lorentz symmetry at high energies comes from the different forms of scaling time and space. This feature generates changes in the action of a free massless scalar field with respect to that one described in Minkowski spacetime. In this sense, the more general action for such a field can be written as

$$S=\int dtd^dx\frac{1}{2}\Bigg[(\partial_t\phi)^2-\sum_{s=1}^{z}g_s\left(\partial^{i_1}\partial^{i_2}\ldots\partial^{i_s}\right)\times\phi\left(\partial_{i_1}\partial_{i_2}\ldots\partial_{i_s}\right)\phi\Bigg]. \qquad (1)$$

The non-perturbative model is that one in which the Eq. (1) only contains the terms where $s=z$ and $g_z=\ell^{2(z-1)}$, where ℓ is a characteristic length. Thus the field equation, according to this model, is given by

$$\partial_t^2\phi-\ell^{2(z-1)}\nabla^{2z}\phi=0. \qquad (2)$$

Considering an oscillatory field of the form $\phi(\vec{x},t)=C\exp(i\vec{k}.\vec{x}-i\varpi t)$, we get the eigenfrequencies given by $\varpi=\ell^{(z-1)}k^z$, where $k=\sqrt{k_x^2+k_y^2+k_z^2}$.

The black body radiation is described by the Stephan–Boltzmann law, which can be found from the Helmholtz free-energy density, expressed by [31]

$$f_{bb}(T)=2k_BT\int\frac{d^3\mathbf{k}}{(2\pi)^3}\log\{(1-e^{-\widetilde{\beta}k^z})\}, \qquad (3)$$

where $\widetilde{\beta}=\ell^{z-1}/k_BT$. The factor 2 was introduced to account for the two modes of polarization of the electromagnetic waves. When $z=1$, namely, in the low-energies domain, this free-energy density is

$$f_{bb}(T)=-\frac{\pi^2(k_BT)^4}{45}, \qquad (4)$$

which leads to the law of Stephan–Boltzmann, from which we obtain the energy density of the black body, given by

$$u_{bb}(T)=-T^2\frac{\partial(f_{bb}/T)}{\partial T}=\frac{\pi^2(k_BT)^4}{15}, \qquad (5)$$

as it should be in the IR limit and in absence of a gravitational field.

On the other hand, if we put $z=3$ in Eq. (3), which corresponds to high-energies scale, the free-energy density will be now given by

$$f_{bb}(T)=-u_{bb}(T)=-\frac{(k_BT)^2}{18\ell^2}. \qquad (6)$$

It is known that the Stephan–Boltzmann law in a d-dimensional flat space yields a black body energy density proportional to T^{d+1} [32,33]. Thus, by the above expression, we can see that the effective dimension of the spacetime at high-energies scales is $d+1=2$, according to the Hořava–Lifshitz theory. This fact will not change when we take into account the gravitational field. Such specific dimensional reduction at UV scale is compatible with other approaches of quantum gravity [5,6]. In fact, this dependence with the square of temperature works for any d-dimensional flat space, provided that $z=d$ in order to have the renormalizability of the theory. In this case, the measure of the integral (3) is $[\pi^{-d/2}2^{-d}d/\Gamma(d/2+1)]k^{d-1}$, where $\Gamma(x)$ is the Gamma function and the black body energy density is exactly given by

$$u_{bb}(T)=\frac{2^{-d}\pi^{(4-d)/2}\ell^{1-d}}{3\Gamma(d/2+1)}(k_BT)^2. \qquad (7)$$

For an arbitrary temperature, a numerical analysis of the above equation reveals that the energy density has a minimum for $d=3$ when $\ell\approx0.2\ell_P$, with ℓ_P being the Planck length, and thus extending the UV domain of the H–L theory to trans-Planckian scales [10].