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Property of various correlation measures of open Dirac system with Hawking effect in Schwarzschild space–time

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ABSTRACT

We explore the performance of various correlation measures for open Dirac system with Hawking effect in Schwarzschild space–time. Our results indicate that the impact of Hawking effect on physical accessible entanglement is weaker than that of decoherence. For generalized amplitude damping (GAD) channel, the entanglement sudden death (ESD) is analyzed in detail, and the inequivalence of quantization for Dirac particles in the black hole and Kruskal space–time is verified via quantum discord measure. In addition, as an example for interpreting Bell non-locality, we study the GAD channel with Hawking effect. It can be noticed that there is a boundary line of Bell violation for physically accessible states. That is, quantum non-locality would disappear when Hawking temperature exceeds a certain value. This critical temperature increases as a decoherence parameter decreases. In the case of phase damping (PD) channel, the interaction between the particle and noise environment does not produce bipartite system–environment entanglement. Then we discuss entanglement distributions, and find that the reduced physically accessible entanglement can be redistributed to physical inaccessible region. At last, we extend our investigation to an N-qubit system, and obtain a universal expression of the physical accessible entanglement.

1. Introduction

Recently, researches about combination of quantum information science and relativity theory have drawn a lot of attention [1–6], since it not only promises a deeper comprehension about quantum mechanics [7] but also offers a new method of understanding the information paradox existing in black hole [8–10]. For instance, in the background of a black hole, Pan et al. [1] discussed the quantum entanglement for scalar field, and Deng et al. [4] studied how the Hawking effect and prepared states could affect entanglement distillability of Dirac fields. More recently, Xu et al. [6] expanded the investigation of the effect of Hawking radiation on multipartite entanglement in Schwarzschild space–time. However, the above investigations are confined to an isolated system for the studies of quantum information. As the real quantum system inevitably suffers from quantum decoherence, this reciprocal interaction between the system and its external noise environment would lead to the degradation of quantum coherence and, in certain cases, produce entanglement sudden death (ESD). It therefore raises the question of how to understand the behaviors of quantum information for open system in Schwarzschild space–time.

To solve the problem, we probe the effect of Hawking radiation [11] on various quantum correlation measures for Dirac particles involved in dissipative environment in Schwarzschild space–time. Our aim is to unveil some interesting phenomena about the characteristic of correlation measurements with Hawking effect and decoherence channel, which may lead to a much better understanding of the Hawking–Unruh effects in quantum information processing. To illustrate this problem properly, we propose the scheme in a situation where two observers, Alice and Bob, share a generically entangled state at the same initial point in the flat Minkowski space–time. During the same time interval, Alice remains at the asymptotically flat region undergoing a decoherence channel, while Bob freely falls in toward a Schwarzschild black hole and eventually locates near the event horizon. Our work here is to provide a better insight into entanglement redistribution and information paradox of the black holes from the perspective of quantum mechanics.

This paper is organized as follows. In Section 2, a brief review of the vacuum structure and Hawking radiation for Dirac fields in Schwarzschild space–time is given. In Section 3, we explore

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the properties of various correlation measures under two different decoherence environments with Hawking effect in the background of a Schwarzschild black hole. Finally, Section 4 summarizes our conclusions.

2. Vacuum structure of Dirac particles in the background of a Schwarzschild black hole

We first introduce a concise review of vacuum structure for Dirac particles in Schwarzschild space–time. The Dirac equation [12] in a curve space–time can be read as

$$\left[\gamma^a e_a^u(\partial_u + \Gamma_u)\right]\psi = 0 \tag{1}$$

while the metric for the Schwarzschild space–time is given by

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{2}$$

Combining Eq. (1) with Eq. (2) then leads to the Dirac equation in the Schwarzschild space–time in the form of

$$-\frac{\gamma_0}{\sqrt{1-\frac{2M}{r}}}\frac{\partial\psi}{\partial t} + \gamma_1\sqrt{1-\frac{2M}{r}}\left[\frac{\partial}{\partial r} + \frac{1}{r} + \frac{M}{2r(r-2M)}\right]\psi + \frac{\gamma_2}{r}\left(\frac{\partial}{\partial\theta} + \frac{\cot\theta}{2}\right)\psi + \frac{\gamma_3}{r\sin\theta}\frac{\partial\psi}{\partial\phi} = 0. \tag{3}$$

For simplicity, G, c, $\hbar$ and k_B are regarded as unity throughout this paper. Then, by solving Eq. (3), we obtain the positive (fermions) frequency outgoing solutions for the outside and inside region near the event horizon [13]

$$\psi_k^{I+} = \Im e^{-i\omega u} \quad (r > r_+),$$
$$\psi_k^{II+} = \Im e^{i\omega u} \quad (r < r_+) \tag{4}$$

where $\Im$ is a 4-component Dirac spinor [14], ω is a monochromatic frequency of the Dirac field, $u = t - r^*$ and $r^* = r + 2M\ln[(r - 2M)/(2M)]$ represent the tortoise coordinate. By using the above complete orthogonal basis (Eqs. (4)), we then quantize the Dirac fields in Schwarzschild space–time

$$\psi_{out} = \sum_\kappa \int dk\left(a_k^\kappa \psi_k^{\kappa+} + b_k^{\kappa+}\psi_k^{\kappa-}\right) \tag{5}$$

where $\kappa = (I, II)$, a_k^κ and $b_k^{\kappa+}$ denote the fermion annihilation and antifermion creation operators, while I and II correspond to the state of exterior and interior region, respectively.

On the other hand, the generalized light-like Kruskal coordinates for the Schwarzschild black hole is introduced as

$$u = -4M\ln\left[U/(4M)\right], \qquad v = 4M\ln\left[V/(4M)\right], \quad \text{if } r < r_+,$$
$$u = -4M\ln\left[-U/(4M)\right], \qquad v = 4M\ln\left[V/(4M)\right], \quad \text{if } r > r_+. \tag{6}$$

And based on the Damour–Ruffini's suggestion [15] we find another complete basis for positive energy modes by making an analytic continuation for Eqs. (4):

$$\Upsilon_k^{I+} = e^{2\pi M\omega_k}\psi_k^{I+} + e^{-2\pi M\omega_k}\psi_{-k}^{II-},$$
$$\Upsilon_k^{II+} = e^{2\pi M\omega_k}\psi_k^{II+} + e^{-2\pi M\omega_k}\psi_{-k}^{I-}. \tag{7}$$

Then the decomposition of the Dirac fields in the Kruskal space–time can be given in terms of these bases:

$$\psi_{out} = \sum_\kappa \int dk\left[2\cosh(4\pi M\omega_k)\right]^{-1/2}\left(c_k^\kappa \Upsilon_k^{\kappa+} + d_k^{\kappa+}\Upsilon_k^{\kappa-}\right) \tag{8}$$

where the c_k^κ and $d_k^{\kappa+}$ represent the annihilation and creation operators in the Kruskal vacuum.

Obviously, these two quantization processes are unequal, and their relationship can be reflected by the Bogoliubov transformations [16] between the creation and annihilation operators in the Schwarzschild and Kruskal coordinates. Given the Bogoliubov relationships being diagonal, each annihilation operator c_k^κ can be represented as

$$c_k^I \to \left(e^{-8\pi M\omega_k} + 1\right)^{-1/2} a_k^I - \left(e^{8\pi M\omega_k} + 1\right)^{-1/2} b_k^{II+}. \tag{9}$$

Through a series of calculation, the vacuum and excited states of the Kruskal particle for mode **k** can be expressed in the basis of Schwarzschild Fock space

$$|0\rangle_K^+ \to \left(e^{-\omega_k/T} + 1\right)^{-1/2} \exp\left[e^{-\omega_k/2T} a_k^{I+} b_{-k}^{II+}\right]|0_k\rangle_I^+ |0_{-k}\rangle_{II}^-$$
$$= \left(e^{-\omega_k/T} + 1\right)^{-1/2}|0_k\rangle_I^+|0_{-k}\rangle_{II}^- + \left(e^{\omega_k/T} + 1\right)^{-1/2}|1_k\rangle_I^+|1_{-k}\rangle_{II}^-,$$
$$|1\rangle_K^+ \to |1_k\rangle_I^+|0_{-k}\rangle_{II}^-, \tag{10}$$

where modes **k** are the spherical harmonics with fixed values of the orbital angular momentum l and the total angular momentum j, $\{|n_k\rangle_I^+\}$ and $\{|n_{-k}\rangle_{II}^-\}$ are the orthonormal bases for the outside and inside regions of the event horizon respectively, the superscript on the kets $\{+, -\}$ denotes the particle and antiparticle vacua, and $T = \frac{1}{8\pi M}$ is the Hawking temperature. Note that states $|0\rangle$ and $|1\rangle$ refer to mode population numbers. For brevity, $\{|n_k\rangle_I^+\}$ and $\{|n_{-k}\rangle_{II}^-\}$ is replaced by $\{|n\rangle_I\}$ and $\{|n\rangle_{II}\}$, respectively.

3. Performance of various correlation measures under noise environments for Dirac particles in Schwarzschild space–time

Assume that Alice stays static with a detector which only detects mode $|n\rangle_A$ in the asymptotically flat region undergoing the influence of decoherence environments, while Bob, with a detector sensitive only to mode $|n\rangle_B$, freely falls toward a Schwarzschild black hole and hovers at a fixed finite nearest distance away from the event horizon. They share a generically entangled state

$$|\phi\rangle = \alpha|00\rangle + \sqrt{1-\alpha^2}|11\rangle, \tag{11}$$

where α is a state parameter that runs from 0 to 1. We then utilize Eq. (10) to rewrite Eq. (11) in terms of Minkowski mode for Alice and black hole mode for Bob. Since the exterior region is causally disconnected from the interior region of the black hole, the physical accessible density matrix ρ_{AB_I} by tracing over the state of the interior region B_{II} can be obtained in the form of

$$\rho_{AB_I} = \alpha^2\mu^2|00\rangle\langle 00| + \alpha^2\upsilon^2|01\rangle\langle 01| + \left(1-\alpha^2\right)|11\rangle\langle 11| + \alpha\mu\sqrt{1-\alpha^2}\left(|00\rangle\langle 11| + |11\rangle\langle 00|\right), \tag{12}$$

where $\mu = (e^{-\omega_k/T} + 1)^{-1/2}$ and $\upsilon = (e^{\omega_k/T} + 1)^{-1/2}$.

3.1. Generalized amplitude damping channel

In this subsection, we discuss the case of Alice under generalized amplitude damping (GAD) channel [17], where the system can both loss and gain excitations by interacting with the environment. Specifically, the spontaneous radiation of a particle exposed to a nonzero temperature can be modeled by the GAD channel. For simplicity, we directly utilize an expedient way, Kraus operators, to