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Brane-world and loop cosmology from a gravity–matter coupling perspective

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ABSTRACT

We show that the effective brane-world and the loop quantum cosmology background expansion histories can be reproduced from a modified gravity perspective in terms of an $f(R)$ gravity action plus a $g(R)$ term non-minimally coupled with the matter Lagrangian. The reconstruction algorithm that we provide depends on a free function of the matter density that must be specified in each case and allows to obtain analytical solutions always. In the simplest cases, the function $f(R)$ is quadratic in the Ricci scalar, R, whereas $g(R)$ is linear. Our approach is compared with recent results in the literature. We show that working in the Palatini formalism there is no need to impose any constraint that keeps the equations second-order, which is a key requirement for the successful implementation of the reconstruction algorithm.

1. Introduction

In the context of the early-time cosmology, the approaches based on brane-world models [1] and loop quantum gravity [2] seem to favour a specific model of cosmic evolution characterized by quadratic relations between the Hubble parameter and the energy density of the fluid, dubbed *quadratic cosmology*. In a recent paper, Bertolami and Páramos [3] considered theories in which, besides an $f(R)$ action for the gravitational field, the matter was also allowed to couple non-minimally to gravity via another function $g(R)$ [4]. They showed that these theories may successfully reproduce any background expansion history. In particular, they found specific forms for the functions $f(R)$ and $g(R)$ able to reproduce quadratic cosmology in a four-dimensional scenario. An interesting aspect of the approach by Bertolami and Páramos is the fact that the expression for the functions $f(R)$ and $g(R)$ can be found analytically and take the simple forms $f(R) = R + \alpha R^2$ and $g(R) = 1 + \beta R$, with α and β being specific constant parameters. The fact that a quadratic $f(R)$ Lagrangian can be directly related with the cosmology of loop quantum cosmology and brane-worlds through the addition of a non-trivial coupling between matter and curvature is remarkable.

The approach followed in [3] to obtain the functions $f(R)$ and $g(R)$ is, however, not fully satisfactory. In fact, as is well known, $f(R)$ theories (with or without matter–curvature couplings) are generically governed by fourth-order field equations, which makes it extremely difficult to find exact solutions. The higher-order derivatives can also be interpreted as representing new dynamical degrees of freedom associated with a scalar field. This scalar field is a function of the Ricci scalar in the case of pure $f(R)$ gravity but also involves the matter Lagrangian in the non-minimally coupled case at hand. In order to avoid the difficulties derived from the existence of higher-order derivatives or new scalar degrees of freedom, the strategy proposed in [3] was to introduce a specific constraint between the functions defining the model in order to remove the new degrees of freedom (or, equivalently, the higher-order derivatives) from the field equations. In our view, this procedure is removing in an ad hoc manner a basic feature and key defining aspect of the theory. In fact, it represents an unnecessary act of violence aimed at forcing a theory to do something that in natural conditions it would not do. Moreover, given that the theory contains new dynamical degrees of freedom by construction, one might expect that small perturbations could excite those degrees of freedom thus making unstable the choice proposed in [3]. Though for some particular models it could be robust, there is no guarantee that this strategy could be valid in general.

In this work, we show that a natural alternative formulation of the problem presented above exists. The key difference between

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our approach and that of Bertolami and Páramos [3] is that we work in the Palatini formalism, i.e., we assume that metric and connection are a priori independent geometrical objects [5]. We find that working in the Palatini formalism, the restriction imposed in [3] is not essential, as the field equations that one obtains are of second-order by construction. We note that the Palatini formulation was also a key element in [6] to obtain an effective action of the $f(R)$ type able to capture the full dynamics of loop quantum cosmology, from the GR limit at low energies to the nonperturbative regime at the bounce. In that approach, curvature–matter couplings were not considered. Here we allow for this possibility and explore its effects and implications.

In the approach of [3], the geometry is implicitly assumed to be Riemannian, i.e., the connection is constrained a priori to be given by the Christoffel symbols of the metric, which is the origin of the higher-order field equations arising in most theories of extended gravity. However, on geometrical grounds, metric and connection are equally fundamental and independent entities, carrying very different geometrical meanings. In this sense, the question on whether the underlying geometry of space–time is Riemannian or otherwise is not a matter of conventions but a foundational issue of gravitational physics that must be answered by experiments. In the case of classical GR, this question is irrelevant because if the connection is taken to be independent, its variation leads to a new equation whose solution is the Levi-Civita connection, thus yielding the same field equations as in the standard (metric) approach. This result follows from the particular functional form of the Einstein–Hilbert Lagrangian but, in general, it does not hold when one moves away from GR (with the main exception of Lovelock theories [7]). When $f(R)$ extensions are considered in the Palatini approach, the connection satisfies a set of algebraic (not differential) equations. The solution can be expressed as the Levi-Civita connection of an auxiliary metric $h_{\mu\nu}$ conformally related with the space–time metric $g_{\mu\nu}$. When non-minimal coupling is allowed, the conformal factor is a function that depends on the Lagrangian $f(R)$, the function $g(R)$, and the matter sources. The resulting equations for the metric are of second-order, like in the minimally coupled case with $g(R)=1$. As a result, the solution to the problem of finding a pair of functions $f(R)$ and $g(R)$ able to reproduce a particular cosmological background history does not require imposing the constraint proposed in [3].

There are several reasons that motivate the study of the matter–curvature coupling within the Palatini framework (see [8] for a discussion on these theories within the metric approach). It is well known from the first recent studies of $f(R)$ theories [9] (see also the review [10]), that the gravitational field in Palatini theories depends intimately on the local energy–momentum *density* distributions, i.e., it is not just determined by the total amounts of energy–momentum in a given region [11,12]. The details of how that energy–momentum is distributed does have an impact on the metric locally. In fact, in simple models of black hole formation, it has been shown by means of exact analytical solutions [13,14] (see also [15] for a perturbative discussion) that the space–time metric not only depends on the total mass of the collapsing fluid but also on the energy density that the fluid carries at each instant of time. When the energy flux that forms the black hole ceases, the resulting geometry only depends on the total accumulated mass. When the flux is on, a dependence on the energy density appears again along the fluid's trajectory. This puts forward that non-trivial interactions between geometry and energy density arise even if one assumes minimal coupling. Furthermore, from the study of scenarios involving the coupling of high-energy extensions of gravity to free electric fields [16], it has been found that point-like particles could be seen as topological entities with wormhole structure [17]. This non-trivial interplay between matter fields and geometry in which particles are seen as microscopic geometric structures [18] naturally motivates the study of curvature–matter couplings as a way to encode high-energy interactions between geometry and topology. Further research to better understand the role and properties of theories with this type of couplings is thus necessary.

In this work we consider a modified $f(R)$ gravity coupled to matter via a function $g(R)$ in the Palatini formalism. We shall explicitly show that the field equations of this setting are always second-order and, in vacuum, boil down to those of GR. This allows to study the gravity–matter coupling framework from a more general perspective than in the standard (metric) approach, since the functions $f(R)$ and $g(R)$ are not forced to satisfy a specific constraint. Here we shall work out this scenario and show that any given cosmological background history can be obtained from a Palatini $f(R)$ theory with gravity–matter couplings. To illustrate this point we will consider the particular case of quadratic cosmology.

2. Palatini theories with gravity–matter coupling

The action defining our theory is written as follows

$$S=\int d^4x\sqrt{-g}\left[\frac{f(R)}{2\kappa^2}+g(R)L_m(\psi_m,g_{\mu\nu})\right], \tag{1}$$

where κ^2 is a constant with suitable dimensions (in GR, $\kappa^2\equiv 8\pi G/c^3$), $\sqrt{-g}$ is the determinant of the space–time metric $g_{\mu\nu}$, $f(R)$ and $g(R)$ are two arbitrary functions of the Ricci scalar $R=g^{\mu\nu}R_{\mu\nu}(\Gamma)$, constructed with the independent connection $\Gamma\equiv\Gamma^{\lambda}_{\mu\nu}$, and L_m is the matter Lagrangian, where ψ_m denotes collectively the matter fields, which are only coupled to the metric for simplicity. To obtain the field equations for the action (1) we perform independent variations with respect to metric and connection (Palatini approach), and further assume vanishing torsion, $\Gamma^{\lambda}_{[\mu\nu]}=0$ [19], which leads to

$$\left(f_R+2\kappa^2 g_R L_m\right)R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}f(R)=\kappa^2 g(R)T_{\mu\nu} \tag{2}$$

$$\nabla^{\Gamma}_{\mu}\left[\sqrt{-g}\left(f_R+2\kappa^2 g_R L_m\right)g^{\alpha\beta}\right]=0, \tag{3}$$

where we have used the shorthand denotation $f_R\equiv df/dR$ and $T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}$ is the energy–momentum tensor of the matter. To solve these equations we note that the connection equations (3) can be solved by introducing a rank-two tensor $h_{\mu\nu}$, related to the metric $g_{\mu\nu}$ as

$$h_{\mu\nu}=\Phi g_{\mu\nu};\qquad h^{\mu\nu}=\frac{1}{\Phi}g^{\mu\nu}, \tag{4}$$

where

$$\Phi\equiv f_R+2\kappa^2 g_R L_m, \tag{5}$$

such that (3) reads $\nabla^{\Gamma}_{\mu}(\sqrt{-h}h^{\alpha\beta})=0$. This implies that the independent connection, $\Gamma^{\lambda}_{\mu\nu}$, becomes the Levi-Civita connection of $h_{\mu\nu}$, which is conformally related to the metric $g_{\mu\nu}$ via (4). Note that the GR case with no matter–curvature coupling corresponds to $f(R)=R$ and $g(R)=1$, which implies that $\Phi=1$ and therefore $g_{\mu\nu}=h_{\mu\nu}$, in agreement with the fact that in this case the action (1) is the standard Einstein–Hilbert action of GR.

On the other hand, tracing with $g_{\mu\nu}$ in (2) we obtain

$$\Phi R-2f(R)=\kappa^2 g(R)T, \tag{6}$$

where $T\equiv T_{\mu}{}^{\mu}$ is the trace of the energy–momentum tensor. This is an algebraic equation that generalizes the linear relation $R=$