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Superconducting loop quantum gravity and the cosmological constant

Stephon H.S. Alexander [a,b], Gianluca Calcagni [a,*]

[a] Institute for Gravitation and the Cosmos, Department of Physics, The Pennsylvania State University, 104 Davey Lab, University Park, PA 16802, USA
[b] Department of Physics, Haverford College, 370 Lancaster Avenue, Haverford, PA 19041, USA

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ABSTRACT

We argue that the cosmological constant is exponentially suppressed in a candidate ground state of loop quantum gravity as a nonperturbative effect of a holographic Fermi-liquid theory living on a two-dimensional spacetime. Ashtekar connection components, corresponding to degenerate gravitational configurations breaking large gauge invariance and CP symmetry, behave as composite fermions that condense as in Bardeen–Cooper–Schrieffer theory of superconductivity. Cooper pairs admit a description as wormholes on a de Sitter boundary.

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If the observed dark energy is associated with all the contributions from quantum fields to the cosmological constant Λ, we have to explain why these are suppressed so as to render the vacuum energy $\Lambda \sim 10^{-120}$ (in reduced Planck units). The dark energy problem is further obscured when the issue of general covariance arises. For example, a zero occupation number vacuum state is not invariant under general coordinate transformations [1]. One approach is to find a mechanism wherein the fundamental degrees of freedom of quantum gravity dynamically regulate the cosmological constant at the level of general covariance. Once a spacetime background is specified, we should be able to identify the root of the ambiguity that we currently face in the evaluation of dark energy.

How does one truly deal with the cosmological constant problem in a background independent manner unless one includes matter fields? Here we entertain the possibility that the vacuum energy evaluated on a degenerate spacetime is responsible for a dynamical suppression of Λ. This sector violates parity and is described, at the quantum level, by a model with a four-fermion interaction which reproduces that of Cooper pairing in Bardeen–Cooper–Schrieffer (BCS) theory [2]. This correspondence is actually more general and it can be shown that gravity allows for a dual description in terms of a nonlocally interacting Fermi liquid; which is done in a companion paper, where the reader will find all the ingredients of this picture reviewed or developed in greater detail [3].

To facilitate the study of the cosmological constant we employ the Ashtekar formalism [4] of classical general relativity, which is described by a complex connection field $A \equiv A_\alpha\, dx^\alpha \equiv A^i_\alpha \tau_i\, dx^\alpha$ and a real triad E obeying a canonical equal-time Poisson algebra $\{A^i_\alpha(\mathbf{x}),\, E^\beta_j(\mathbf{y})\} = i\delta^\beta_\alpha \delta^i_j \delta(\mathbf{x},\mathbf{y})$, where Greek indices $\alpha, \beta, \ldots$ denote spatial components, Latin letters $i, j, \ldots$ label directions in the internal gauge space, and τ_i are generators of the $su(2)$ gauge algebra. Introducing the gauge field strength $F^k_{\alpha\beta} \equiv \partial_\alpha A^k_\beta - \partial_\beta A^k_\alpha + \epsilon_{ij}{}^k A^i_\alpha A^j_\beta$ and the magnetic field $B^{\alpha i} \equiv \epsilon^{\alpha\beta\gamma} F^i_{\beta\gamma}/2$ ($\epsilon_{\alpha\beta\gamma}$ is the Levi-Civita symbol), the scalar, Gauss, and vector gravitational constraints in the presence of a cosmological constant are

$$\mathcal{H} = \epsilon_{ijk} E^i \cdot E^j \times \left(B^k + \frac{\Lambda}{3} E^k\right) = 0,$$

$$\mathcal{G}_i = D_\alpha E^\alpha_i = 0,$$

$$\mathcal{V}_\alpha = (E_i \times B^i)_\alpha = 0,$$

where $(\mathbf{a} \times \mathbf{b})_\alpha = \epsilon_{\alpha\beta\gamma} a^\beta b^\gamma$ and D_α is the covariant derivative. The Gauss constraint guarantees invariance under gauge transformations homotopic to the identity; the total Hamiltonian can be made invariant also under large gauge transformations by shifting the momentum E by the axial magnetic field [5,6].

A candidate background-independent quantum theory of gravity at small scales is loop quantum gravity (LQG) [7], which will provide the necessary interpretational framework. The triad becomes the operator $\hat{E}^\alpha_i = -\delta/\delta A^i_\alpha$, while $\hat{A}^i_\alpha$ is multiplicative in

* Corresponding author.
E-mail addresses: stephonalexander@mac.com (S.H.S. Alexander), gianluca@gravity.psu.edu (G. Calcagni).

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the naive connection representation. The quantum constraints on a kinematical state $\Psi(A)$ read

$$\hat{\mathcal{H}}\Psi(A) = \epsilon_{ijl}\epsilon_{\alpha\beta\gamma}\frac{\delta}{\delta A_{\alpha i}}\frac{\delta}{\delta A_{\beta j}}\hat{\mathcal{S}}^{\gamma l}\Psi(A) = 0, \tag{1}$$

$$\hat{\mathcal{G}}_i\Psi(A) = -D_\alpha\frac{\delta}{\delta A_{\alpha i}}\Psi(A) = 0, \tag{2}$$

$$\hat{\mathcal{V}}_\alpha\Psi(A) = -\frac{\delta}{\delta A^i_\beta}F^i_{\alpha\beta}\Psi(A) = 0, \tag{3}$$

where $\mathcal{S}^{\gamma l} \equiv B^{\gamma l} + (2k/\pi)E^{\gamma l}$ and

$$k \equiv \frac{6\pi}{\Lambda} \tag{4}$$

in Lorentzian spacetime, and we adopted a factor ordering with the triads on the left [4,8], which makes the quantum constraint algebra consistent. A solution to all constraints (in their smeared form) is the Chern–Simons state [9] (see [10,11] for reviews)

$$\Psi_{\rm CS} = \mathcal{N}\exp\left(\frac{k}{4\pi}\int_{S^3} Y_{\rm CS}\right), \tag{5}$$

where $\mathcal{N}$ is a normalization constant and $Y_{\rm CS}$ is the Chern–Simons form

$$Y_{\rm CS} \equiv \frac{1}{2}{\rm tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right), \tag{6}$$

on the 3-sphere. Solution of the Hamiltonian constraint is guaranteed by the property

$$\frac{\delta}{\delta A_{\gamma k}}\int_{S^3} Y_{\rm CS} = 2B^{\gamma k}.$$

Non-Abelian gauge theories like QCD made invariant under large gauge transformations admit instantonic degrees of freedom that have phenomenological consequences [12,13]. Likewise, the Chern–Simons ground state encodes degrees of freedom connecting families of spacetimes. In Euclidean gravity ($k \equiv i\theta/\pi \to i6\pi/\Lambda$), different sectors are connected by unitary large gauge transformations which shift the topological phase θ by integer values. Under a large gauge transformation characterized by winding number n, $\int Y_{\rm CS} \to \int Y_{\rm CS} + 4\pi^2 n$ [14], and the Chern–Simons state transforms as $\Psi_{\rm CS}(A) \to e^{in\theta}\Psi_{\rm CS}(A)$ [5].

The inclusion of matter perturbations reproduce standard quantum field theory on de Sitter background [15], while linearizing the quantum theory one recovers long-wavelength gravitons on de Sitter [10]. In this sense the Chern–Simons state is a genuine ground state of the theory. When the connection is real, the Chern–Simons state closely resembles a topological invariant of knot theory [16]. This identification opens up a possibility of describing quantum gravitational dynamics with the mathematics developed in knot theory [17]. Also, it is generally believed that matter excitations might be realized as particular states in the spin network space, where braid configurations (corresponding to standard model generations) are expected to live [18].

Despite these and other beautiful properties, several issues have been raised against the Chern–Simons state, including the problem of normalizability ($\Psi_{\rm CS}$ is not normalizable), reality (A is self-dual), and representation (we still lack a suitable measure in configuration space allowing one to resort to the LQG holonomy representation). Nevertheless, the Chern–Simons state has passed several independent consistency checks [11], and many of the above objections have been addressed at least partly in recent investigations [19,20].

Whenever a cosmological constant component is required by observations at some point during the evolution of the universe, agreement with physical observables lead to the phenomenological hypothesis that the vacuum component is actually dynamical; its behaviour can be reproduced by the matter (typically, scalar) field characteristic of quintessence and inflationary models. In this respect what we are going to do, promoting Λ to an evolving functional $\Lambda(A)$, is not uncommon. Nonetheless, the justification and consequences of this step change perspective under the lens of loop quantum gravity. Looking at Eq. (4), one can see that a dynamical cosmological constant corresponds to a deformation of the topological sector of the quantum theory:

$$k \to k(A). \tag{7}$$

In QCD the partition function can be extended to a larger $U(1)$ symmetry, namely the Peccei–Quinn invariance under a rotation by the θ angle [21]. Instanton effects can spontaneously break the $U(1)$ symmetry resulting in a light particle called axion. Likewise, a large gauge transformation in Chern–Simons wavefunction is regarded as a $U(1)$ rotation, so by deforming θ we break this symmetry. Classically, unless k is invariant under small gauge transformations, the only sectors compatible with Eq. (7) and the Gauss constraint are degenerate ($\det E = 0$). We exclude the most degenerate case ${\rm rk}\,E = 0$, as we want to preserve at least part of the canonical algebra. This leaves the cases ${\rm rk}\,E = 1, 2$.

The deformed quantum scalar constraint is defined with $\Lambda(A)$ to the left of triad operators; however, if the triad operator is degenerate the scalar constraint plays no dynamical role. Therefore it is consistent to assume the same attitude as in the discussion of the Chern–Simons state, and require that the deformed state Ψ_* annihilates the deformed reduced constraint $\mathcal{S}^{\alpha i}_*$. The latter requires the addition of a counterterm,

$$\left(\hat{\Theta}^{\alpha i} + \hat{\mathcal{S}}^{\alpha i}_*\right)\Psi_* = \hat{\Theta}^{\alpha i} + \frac{1}{2}\int_{S^3} Y_{\rm CS}\frac{\delta\ln\Lambda(A)}{\delta A_{\alpha i}} = 0, \tag{8}$$

which breaks large gauge invariance. (Nonlocal effects, expected from this symmetry breaking, will be discussed later.) The equation of motion for the gauge field becomes

$$\dot{A}^i_\alpha = i\epsilon^i{}_{jk}E^{\beta j}\left[F^k_{\alpha\beta} + \frac{\Lambda}{2}\epsilon_{\alpha\beta\gamma}E^{\gamma k} - \epsilon_{\alpha\beta\gamma}\int_{S^3} Y_{\rm CS}\frac{\delta\ln\Lambda(A)}{\delta A_{\gamma k}}\right]. \tag{9}$$

We now make a crucial connection which was demonstrated by Jacobson for classical gravity [22]. One of the advantages of the formulation of classical gravity via Ashtekar variables is the possibility to have a well-defined causal structure [23] even when the background metric is singular, as inside a black hole, at big-bang or big-crunch events, or in processes where topology changes. In particular, the degenerate sector with ${\rm rk}\,E = 1$ describes a two-dimensional spacetime whose future is a tipless wedge. The gravi-electric line can be chosen to lie on the z direction; after some other gauge fixing, Jacobson's sector read

$$A^i_z = 0, \qquad A^3_a = A^3_a(x^a), \qquad A^{i\neq 3}_a = A^{i\neq 3}_a(t,z), \tag{10a}$$

$$E^z_{i\neq 3} = 0, \qquad E^a_i = 0, \qquad E^z_3 = 1. \tag{10b}$$

The classical equation of motion for the transverse–transverse components of the connection is $\dot{A}^i_a = -i\epsilon^{i3}{}_j\partial_z A^j_a$, which can be written as the $(1+1)$-dimensional Dirac equation

$$\gamma^0\dot{\psi} + \gamma^z\partial_z\psi = 0, \tag{11}$$

where γ^μ are Dirac matrices and

$$\psi \equiv \begin{pmatrix} iA^1_1 \\ A^1_2 \\ A^2_1 \\ iA^2_2 \end{pmatrix}. \tag{12}$$

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