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Constraining axion dark matter with Big Bang Nucleosynthesis

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ABSTRACT

We show that Big Bang Nucleosynthesis (BBN) significantly constrains axion-like dark matter. The axion acts like an oscillating QCD θ angle that redshifts in the early Universe, increasing the neutron–proton mass difference at neutron freeze-out. An axion-like particle that couples too strongly to QCD results in the underproduction of ^{4}He during BBN and is thus excluded. The BBN bound overlaps with much of the parameter space that would be covered by proposed searches for a time-varying neutron EDM. The QCD axion does not couple strongly enough to affect BBN.

The axion is a well-motivated dark-mater (DM) candidate that can arise in a variety of models [1]. The allowed mass of these light scalars is relatively unconstrained, spanning many orders of magnitude. Identifying the regions of axion parameter space that are excluded by cosmological and astrophysical constraints is of the utmost importance as it directs the focus of laboratory searches. This Letter presents a new constraint on axion dark matter arising from Big Bang Nucleosynthesis.

The axion was originally introduced to explain why the QCD θ term,

$$S = \frac{\theta}{4\pi^2}\int \mathrm{tr}\, G \wedge G, \tag{1}$$

is not realized in Nature, often referred to as the strong CP problem [2–4].[1] The QCD θ term in (1) induces a neutron electric dipole moment (EDM) $d_n \approx 2.4 \times 10^{-16}\theta$ e cm [5] that is in tension with experiment for $\theta > 10^{-10}$ [6,7]. The axion solves this problem by promoting the parameter θ to a dynamical field, $\theta \to (a/f_a)$, whose potential is minimized at $a = 0$.

The axion is often assumed to be the pseudo-Goldstone boson of a $U(1)$ PQ symmetry, which is spontaneously broken at some high scale, f_a [3,4,8,9]. For the axion to solve the strong CP problem, the explicit breaking of the PQ symmetry must be absent to very high accuracy in the UV [10,11]. The leading potential that the axion is allowed to receive should come from the QCD chiral anomaly. The QCD instantons break the PQ symmetry explicitly, and in the presence of bare quark masses, the axion picks up a mass [3,4]

$$f_a m_a = f_\pi m_\pi \frac{\sqrt{m_u m_d}}{m_u + m_d}, \tag{2}$$

where $m_\pi \approx 140$ MeV is the pion mass, $f_\pi \approx 92$ MeV is the decay constant, and $m_u \approx 2.3$ MeV ($m_d \approx 4.8$ MeV) is the mass of the up (down) quark.

The cosmological equation of state of axion DM is governed by the classical oscillations of the background field [12–15]:

$$a(t) = a_0 \cos(m_a t) = \frac{\sqrt{2\rho_{\mathrm{DM}}}}{m_a}\cos(m_a t). \tag{3}$$

The amplitude a_0 is fixed by requiring that the axion makes up the observed dark matter density, ρ_{DM}. A parameter space spanning orders of magnitude in m_a and f_a is available to axion DM. Constraints on axions that come from their coupling to $G \wedge G$ arise from excess cooling of SN 1987A [16,17] and from static neutron EDM measurements [6,7,16]. Axions may also be constrained through their coupling to $E \cdot B$ (see [1] for a review).

In addition to the QCD axion, axion-like particles (ALPs) can arise in many models. ALPs do not necessarily couple to $G \wedge G$; for example, they may only couple to electromagnetism through the operator $E \cdot B$. In these models, (2) may be violated, and in particular, it is possible that

$$f_a m_a \ll \Lambda_{\mathrm{QCD}}^2. \tag{4}$$

From this point forward, we will use "axion" to refer to both the standard QCD axion and ALPs that couple to $G \wedge G$ with coupling $\propto f_a^{-1}$ and that satisfy (4).

Axions that couple to $G \wedge G$ and simultaneously satisfy (4) may be tested directly in the near future by proposed laboratory searches for an oscillating axion-induced nucleon EDM [16,18,19]. This Letter focuses on this region of axion parameter space.

[1] Our conventions are $\int \mathrm{tr}\, G \wedge G = (1/4)\int d^4x \epsilon^{\mu\nu\alpha\beta}\, \mathrm{tr}\, G_{\mu\nu} G_{\alpha\beta}$, where $G = (1/2) G_{\mu\nu} dx^\mu \wedge dx^\nu$ is the gluon field-strength with the trace taken over gauge indices. In the following, we use also $\tilde{G}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$.

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First, we use chiral perturbation theory (ChPT) to show that the presence of an axion-induced nucleon EDM is in tension with (4) because the axion contribution to the nucleon EDM is associated with the irreducible QCD contribution to the axion mass in (2). As far as we know, the only way to avoid this minimum axion mass is to invoke fine-tuned cancellations, exacerbating the strong CP problem. In particular, we show in Appendix A that it is not possible to reach the parameter space (4) by invoking mixing between multiple axion states. Contrarily, $f_a m_a \gg \Lambda^2_{\rm QCD}$ is possible without fine tuning through axion mixing.

Even if one is willing to ignore fine-tuning arguments, Big Bang Nucleosynthesis (BBN) provides a strong observational constraint. The constraint arises from two simple observations. First, the QCD θ term leads to a shift in the neutron–proton mass difference, as pointed out in [20]. This nuclear mass difference is again dictated by ChPT and is directly related to the axion-induced EDM. Second, the effective θ term induced by axion DM redshifts in the early Universe, roughly as $\theta \sim (1+z)^{3/2}$. Thus, while the effect of axion DM on the neutron–proton mass difference today seems unobservably small, it can be large enough to disturb the production of light elements at the time of BBN ($z \sim 10^{10}$).

We begin by recalling the results from ChPT that relate the axion mass and some of its couplings. Considering only the axion and strongly-interacting SM fields just above the QCD scale, the most general effective Lagrangian that connects the axion to the SM and respects the axion shift symmetry is

$$\mathcal{L} = -\frac{a}{f_a}\frac{G^a_{\mu\nu}\tilde{G}^{a\mu\nu}}{32\pi^2} - \frac{\partial_\mu a}{f_a}\sum_\psi c_\psi \bar{\psi}\bar{\sigma}^\mu \psi \tag{5}$$

to leading order in f_a^{-1} [21]. The left-handed Weyl spinors ψ include u, u^c, d, d^c etc. The coefficients c_ψ are model-dependent. In general they can be off-diagonal in flavor space, but this does not affect the following discussion.

Below the QCD scale, (5) is translated to the chiral Lagrangian, and the axion couplings with pions and nucleons may be computed from ordinary ChPT. The axion enters into the chiral Lagrangian only through the quark mass spurion and through mixed derivative couplings with the neutral pion. Working in the physical basis after diagonalizing the axion–pion mass matrix and kinetic terms, we are particularly interested in the following terms in the chiral Lagrangian:

$$\begin{aligned}\mathcal{L} \supset &-\frac{1}{2}\frac{f_\pi^2 m_\pi^2 m_u m_d}{(m_u+m_d)^2}\left(\frac{a}{f_a}\right)^2 \\ &- \bar{N}\pi\cdot\sigma\left(i\gamma^5 g_{\pi NN} - 2\bar{g}_{\pi NN}\frac{a}{f_a}\right)N \\ &+ \frac{f_\pi \bar{g}_{\pi NN}}{2}\frac{m_d-m_u}{m_d+m_u}\left(\frac{a}{f_a}\right)^2 \bar{N}\sigma^3 N.\end{aligned} \tag{6}$$

Here $N = \binom{p}{n}$ are the nucleons, and the numerical couplings are $g_{\pi NN} \approx 13.5$ and $\bar{g}_{\pi NN} \approx \frac{m_u m_d}{m_u+m_d}\frac{2(M_\Xi - M_\Sigma)}{(2m_s - m_u - m_d)f_\pi} \approx 0.023$ [20,22].

The first line in (6) is the irreducible contribution to the axion mass quoted in (2). We know of no way to eliminate this contribution for an axion with decay constant f_a besides to cancel it with some unrelated mass correction associated with some new Lagrangian term $\Delta\mathcal{L}(a) \propto \delta m^2 (a+\delta\theta)^2$. Such a cancelation would involve fine-tuning the parameter δm^2 by an amount

$$\Delta_{\rm mass} \sim \frac{f_a^2 m_a^2}{f_\pi^2 m_\pi^2} \sim 10^{-14}\left(\frac{f_a m_a}{10^{-9}\ {\rm GeV}^2}\right)^2. \tag{7}$$

Moreover, $\delta\theta$ must also be tuned to avoid CP violation, thereby restoring the strong CP problem on top of the mass fine-tuning in (7). We comment that it is not possible to reach the parameter space (4) by introducing multiple axion-like states and invoking mixing between them, as might be conceived in some string-inspired models [23] (see Appendix A).

The second line in (6) gives the dominant contribution to the axion-induced neutron EDM [18,22],

$$d_n \approx \left(\frac{a}{f_a}\right)\frac{e g_{\pi NN}\bar{g}_{\pi NN}}{4\pi^2}\frac{\ln(4\pi f_\pi/m_\pi)}{m_n}, \tag{8}$$

with m_n the neutron mass.

The third line in (6) gives the axion-induced neutron–proton mass splitting,

$$\begin{aligned} m_n - m_p &= Q_0 + \delta Q, \\ \delta Q &\approx \frac{f_\pi \bar{g}_{\pi NN}}{2}\left(\frac{m_d-m_u}{m_d+m_u}\right)\left(\frac{a}{f_a}\right)^2 \\ &\approx (0.37\ {\rm MeV})\left(\frac{a}{f_a}\right)^2, \end{aligned} \tag{9}$$

when evaluated on a classical axion-field background. $Q_0 \approx 1.293$ MeV is the measured mass difference between the neutron and proton. Thus, an axion field that induces a nuclear EDM also affects the neutron–proton mass splitting in a directly related way. Moreover, the relation between the two effects does not depend on the model-dependent c_ψ coefficients, to leading order in $1/f_a$.[2] We now explore the consequence of the shift in the nuclear mass difference on nucleosynthesis.

For $m_a \gg H(z)$, where $H(z)$ is the proper Hubble expansion rate at redshift z, the axion DM may be treated as an ensemble of Bose–Einstein condensed non-relativistic particles [15]. Neglecting any temperature dependence in m_a, the time-dependent effective θ angle in this limit is

$$\begin{aligned}\theta_{\rm eff}(t) &= \left(1+z(t)\right)^{3/2}\frac{\sqrt{2\bar{\rho}_{\rm DM}}}{f_a m_a}\cos(m_a t) \\ &\approx 5\times 10^{-9}\left(\frac{{\rm GeV}^2}{f_a m_a}\right)\left(\frac{1+z(t)}{10^{10}}\right)^{3/2}\cos(m_a t),\end{aligned} \tag{10}$$

where $\bar{\rho}_{\rm DM} \approx 2.7\times 10^{-27}\ {\rm kg/m^3}$ is the mean cosmological DM energy density today [24]. Neutron freeze-out occurs at temperatures of order 1 MeV, meaning that (10) is adequate for calculating a BBN bound as long as $m_a \gg (1\ {\rm MeV})^2/m_{\rm pl} \approx 10^{-16}$ eV. We begin by discussing m_a in this regime and extend the calculation to the ultra-light regime, $m_a \ll 10^{-16}$ eV, later.

Substituting (10) into (9) shows that axion DM increases the mass difference between the neutron and proton at BBN. This reduces the relative occupation number of neutrons compared to that of protons in thermal equilibrium just before neutron freeze-out, reducing the resulting mass fraction, Y_p, of ^{4}He. The net effect is stronger at smaller $f_a m_a$. We now provide an analytic estimate of the dependence of Y_p on $f_a m_a$, subsequently moving on to a more precise numerical calculation.

After the quark–hadron transition, neutrons and protons are kept in equilibrium through the weak interactions

$$\begin{aligned} &n \longleftrightarrow p + e^- + \bar{\nu}_e, \\ &\nu_e + n \longleftrightarrow p + e^-, \\ &e^+ + n \longleftrightarrow p + \bar{\nu}_e. \end{aligned} \tag{11}$$

[2] The relation between the nuclear EDM and the neutron–proton mass splitting could be modified if we allow for other sources of explicit PQ symmetry breaking beyond the mass-tuning term. We do not consider such possibilities in this Letter.