



Electron $g-2$ in Light-front Quantization



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ABSTRACT

Basis Light-front Quantization has been proposed as a nonperturbative framework for solving quantum field theory. We apply this approach to Quantum Electrodynamics and explicitly solve for the light-front wave function of a physical electron. Based on the resulting light-front wave function, we evaluate the electron anomalous magnetic moment. Nonperturbative mass renormalization is performed. Upon extrapolation to the infinite basis limit our numerical results agree with the Schwinger result obtained in perturbation theory to an accuracy of 0.06%.

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1. Introduction

Nonperturbative approaches in quantum field theory are needed for many applications. One important application is to study bound state problems in strongly interacting systems, *e.g.*, solving for the hadron structure in Quantum Chromodynamics (QCD). The Basis Light-front Quantization (BLFQ) approach has recently been constructed [1,2] as a nonperturbative framework for quantum field theory in the Hamiltonian framework [3]. In previous work [2] this method has been applied to QED for the anomalous magnetic moment of a physical electron in a cavity formed by an external transverse harmonic-oscillator potential, which acts as a regulator of QED dynamics. In addition, the extension to strong time-dependent external field applications (tBLFQ) has been developed and successfully applied [4,5].

In this work we employ BLFQ to compute the electron anomalous magnetic moment in vacuum,¹ which was first calculated in perturbation theory in Ref. [7]. For an alternative nonperturbative calculation in light-front dynamics, see Refs. [8,9].

2. Light-front Hamiltonian in BLFQ basis

In light-front dynamics, physical processes are described in terms of light-front coordinates, which consist of light-front time $x^+ \equiv x^0 + x^3$, the longitudinal direction $x^- = x^0 - x^3$ as well as the transverse coordinates $x^\perp = \{x^1, x^2\}$. We begin with the light-front QED Hamiltonian which can be derived from the ordinary Lagrangian through the standard Legendre transform with the adoption of the light-cone gauge ($A^+ = 0$), where the photon field has physical polarization and positive metric. The resulting QED light-front Hamiltonian takes the following form,

$$P^- = \int d^2x^\perp dx^- \left[\frac{1}{2} \bar{\psi} \gamma^+ \frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+} \psi + \frac{1}{2} A^j (i\partial^\perp)^2 A^j + e j^\mu A_\mu + \frac{e^2}{2} j^+ \frac{1}{(i\partial^+)^2} j^+ + \frac{e^2}{2} \bar{\psi} \gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+} \gamma^\nu A_\nu \psi \right], \quad (1)$$

where Ψ and A_μ are the fermion and gauge boson fields, respectively. The first and second terms are their corresponding kinetic energy terms, and the remaining three terms describe the interaction between the fermion and gauge boson fields. Specifically, these are the *vertex interaction*, the *instantaneous-photon interaction* and the *instantaneous-fermion interaction* in order of appearance in Eq. (1). The m_e and e are the bare electron mass and the bare electromagnetic coupling constant, respectively. In this work we only keep $|e\rangle$ and $|\gamma\rangle$; *i.e.*, two Fock sectors, in our basis (see below). Consequently, the instantaneous-photon interaction does not

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¹ In addition, several improvements and corrections are carried out over Ref. [2]: 1) we correct the operator used in [2] for extracting the anomalous magnetic moment by adopting the standard Pauli form factor operator [6]; 2) we correct a numerical error in [2], which led to an overestimate of the interaction terms in QED Light-front Hamiltonian by a factor of 2; 3) we optimize the computational efficiency on both the analytic and coding level and implement parallel computing so that the BLFQ calculation can take advantage of currently available supercomputers.

contribute since it involves Fock sectors with one more electron (or positron). Moreover, the instantaneous fermion interaction either contributes to overall renormalization factors, which do not affect the intrinsic structure of the physical electron, or contains small- x divergences which need to be cancelled by explicit fermion exchange contributions from higher Fock-sectors. Thus, we defer the inclusion of the instantaneous interactions and adopt the following Hamiltonian for this work,²

$$P^- = \int d^2x^\perp dx^- \left[\frac{1}{2} \bar{\psi} \gamma^+ \frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+} \psi + \frac{1}{2} A^j (i\partial^\perp)^2 A^j + e j^\mu A_\mu \right]. \quad (2)$$

In the second step we construct the Fock-sector basis expansion. A physical electron, which is the focus of this work, receives contributions from the multiple Fock-sectors,

$$|e_{\text{phys}}\rangle = a|e\rangle + b|e\gamma\rangle + c|e\gamma\gamma\rangle + d|ee\bar{e}\rangle + \dots, \quad (3)$$

and each Fock-sector itself consists of an infinite number of basis states. For the purpose of numerical calculations we adopt both a Fock-sector truncation and limits on the basis states in each sector. In this work we make the lowest nontrivial truncation by retaining the $|e\rangle$ and $|e\gamma\rangle$ Fock-sectors only. This is sufficient for obtaining the (nonperturbative) electron wave function accurate to the leading order of the electromagnetic coupling constant $\alpha = \frac{e^2}{4\pi}$.

For each Fock-particle we employ a 2D-harmonic oscillator (HO) wave function, $\Phi_{nm}(p^\perp)$, to describe its transverse degrees of freedom and a plane-wave, $e^{-ip^+x^-/2}$, to describe its longitudinal motion. For each Fock-particle the (transverse) 2D-HO wave function carries the radial quantum number n and angular quantum number m , (as well as a parameter b , setting the scale of the HO wave functions, e.g., in $\exp[-(p^\perp)^2/(2b^2)]$). The (longitudinal) plane-wave carries one quantum number, $k = p^+L/(2\pi)$, which is proportional to the longitudinal momentum p^+ , and L is the length of the longitudinal “box” in which we compactify the longitudinal degrees of freedom of the system. With the additional quantum number λ for the helicity, 4 quantum numbers label each single particle state.

In the transverse directions we implement the “ N_{max} ” truncation in analogy with the 3D-HO truncation in nuclear many-body theory [11,12]. Define a sum, N_β , over the HO quantum numbers for all Fock particles, i , in a specific basis state (here we discuss only the quantum numbers of the transverse spatial motion), $|\beta\rangle$, according to,

$$N_\beta \equiv \sum_i 2n_i + |m_i| + 1. \quad (4)$$

We truncate the basis states by eliminating states with N_β larger than a chosen cutoff N_{max} . Increasing N_{max} not only enhances the resolution but also provides a higher ultraviolet cutoff and a lower infrared cutoff for the particles’ transverse motion.

In the longitudinal direction we perform basis truncation by imposing (anti)periodic boundary condition for (fermions) bosons, such that the longitudinal momentum quantum number k for each Fock particle can only take (half-)integers. Being a good quantum number for the QED Hamiltonian, the total longitudinal momentum summed over all Fock particles in a basis state, $P^+ = \sum_i p_i^+ \propto K$, acts as an additional cutoff. Larger $P^+(K)$ allows more

possible partitions of longitudinal momentum among Fock particles in a basis state and thus leads to a higher resolution in the longitudinal degrees of freedom.

Therefore, in order to specify the truncated basis, we need the following information: 1) Fock-sectors included; 2) truncation parameters, N_{max} (transverse) and K (longitudinal); 3) 2D-HO basis parameter b . The longitudinal period L is not needed due to the longitudinal boost-invariance of light-front dynamics: the light-front wave functions only depend on the longitudinal momentum fraction $x_i = p_i^+/P^+ = k_i/K$. In this work we retain only $|e\rangle$ and $|e\gamma\rangle$ Fock-sectors, and compare numerical results evaluated in bases of different N_{max} and K . Although in the $N_{\text{max}} \rightarrow \infty$ limit, results should be independent of b , we choose $b = M = 0.511$ MeV as the natural value for calculations at finite N_{max} . The (in-) dependence of b in the numerical results for the electron anomalous magnetic moment a_e , around this value of b , will be checked in Section 4.

Next we express our field operators in the BLFQ basis, specifically for the fermion and gauge boson field,

$$\psi(x) = \sum_{\bar{\beta}} \frac{1}{\sqrt{2L}} \int \frac{d^2p^\perp}{(2\pi)^2} [b_{\bar{\beta}} \Phi_{nm}(p^\perp) u(p, \lambda) e^{-ip \cdot x} + d_{\bar{\beta}}^\dagger \Phi_{nm}^*(p^\perp) v(p, \lambda) e^{ip \cdot x}], \quad (5)$$

$$A_\mu(x) = \sum_{\bar{\beta}} \frac{1}{\sqrt{2Lp^+}} \int \frac{d^2p^\perp}{(2\pi)^2} [a_{\bar{\beta}} \Phi_{nm}(p^\perp) \epsilon_\mu(p, \lambda) e^{-ip \cdot x} + a_{\bar{\beta}}^\dagger \Phi_{nm}^*(p^\perp) \epsilon_\mu^*(p, \lambda) e^{ip \cdot x}], \quad (6)$$

where the u and v are the Dirac spinors for fermions and anti-fermions, respectively. The ϵ_μ is the photon polarization vector. The $p \cdot x = p^+x^-/2 - p^\perp x^\perp$ is the inner product between the 3-momentum $p = \{p^+, p^\perp\}$ and the coordinate $x = \{x^-, x^\perp\}$. The $b_{\bar{\beta}}^\dagger$, $d_{\bar{\beta}}^\dagger$ and $a_{\bar{\beta}}^\dagger$ are creation operators for the fermion, antifermion and gauge boson fields, respectively, with quantum numbers $\bar{\beta} = \{k, n, m, \lambda\}$. They satisfy the (anti)commutation relations

$$\{b_{\bar{\beta}}, b_{\bar{\beta}'}^\dagger\} = \{d_{\bar{\beta}}, d_{\bar{\beta}'}^\dagger\} = [a_{\bar{\beta}}, a_{\bar{\beta}'}^\dagger] = \delta_{\bar{\beta}\bar{\beta}'}. \quad (7)$$

Through Eqs. (2) and (7), we are able to write down the light-front QED Hamiltonian in the BLFQ basis. Since we are interested in the mass eigenspectrum contributed by the intrinsic rather than center-of-mass motion, we add an appropriate Lagrange multiplier term to the input light-front QED Hamiltonian. This has the effect of shifting the states with excited center-of-mass motion to high mass and the low-lying spectrum comprises states with lowest center-of-mass motion, following the techniques of nuclear many-body theory. The resulting low-lying states can be written as a simple product of internal and center-of-mass motion [11,12] (see [13] for more details).

Upon diagonalization of the resulting sparse Hamiltonian matrix, one obtains its eigenvalue spectrum and corresponding eigenvectors. In this work, the ground state of the Hamiltonian, with net fermion number being one ($n_f = 1$), is identified as the physical electron. Its eigenvalue, P_e^- , gives the electron mass according to $M^2 \equiv P_e^- P_e^+ - P_\perp^2$, where $P_\perp$ is the total transverse momentum operator.

3. Renormalization

Before we are ready to obtain the electron wave function, one more technical detail needs to be worked out: renormalization. In BLFQ, a nonperturbative approach, the renormalization procedures are different from those adopted in perturbation theory [14].

² The instantaneous fermion or photon interaction can also form a contact interaction in the single-electron sector, as mentioned in Ref. [10]. This type of interactions can, however, be absorbed into the fermion mass counterterm and does not need to be included explicitly in the Hamiltonian (see Section 3).

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