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Diagrammatic insights into next-to-soft corrections

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ABSTRACT

We confirm recently proposed theorems for the structure of next-to-soft corrections in gauge and gravity theories using diagrammatic techniques, first developed for use in QCD phenomenology. Our aim is to provide a useful alternative insight into the next-to-soft theorems, including tools that may be useful for further study. We also shed light on a recently observed double copy relation between next-to-soft corrections in the gauge and gravity cases.

1. Introduction

It is well-known that scattering amplitudes in gauge and gravity theories contain infrared divergences. These arise from the emission of *soft* gluons or gravitons, whose 4-momentum tends to zero. The remaining *hard* particles in the amplitude are then said to obey the *eikonal approximation*, and it can be shown that amplitudes factorise in this limit. At tree level, for example, the amplitude for the emission of n hard gluons (momenta $\{p_i\}$) and one soft gluon (momentum k) can be written as

$$\mathcal{A}_{n+1}(\{p_i\},k)=\mathcal{S}_n^{(0)}\mathcal{A}_n(\{p_i\});\qquad \mathcal{S}_n^{(0)}=\sum_{i=1}^{n}\frac{\epsilon_\mu(k)p_i^\mu}{p_i\cdot k},\tag{1}$$

where we neglect the coupling constant and colour factors of the soft emission for brevity. Here $\mathcal{A}_n$ is the amplitude for the n hard particles with no additional emission, and $\epsilon_\mu(k)$ is the polarisation vector of the soft gluon. The gravity equivalent of this is known as Weinberg's soft theorem [1], and takes the form

$$\mathcal{M}_{n+1}(\{p_i\},k)=\mathcal{S}_{n,\mathrm{grav.}}^{(0)}\mathcal{M}_n(\{p_i\});$$
$$\mathcal{S}_{n,\mathrm{grav.}}^{(0)}=\sum_{i=1}^{n}\frac{\epsilon_{\mu\nu}(k)p_i^\mu p_i^\nu}{p_i\cdot k}.\tag{2}$$

Until recently, much less has been known about the corrections to Eqs. (1), (2), upon performing a systematic expansion in the momentum of the soft gauge boson. Such corrections are known as next-to-soft, and the hard emitting particles then obey the *next-to-eikonal* approximation. The phenomenological impact of such corrections has been studied in QCD [2–6], and a systematic attempt to classify them has been made in [7–9]. The gravitational consequences of next-to-soft radiation have been explored in [10].

An orthogonal recent body of work has explored such contributions from a more formal point of view. Based on the observation that Weinberg's soft theorem can be interpreted as a Ward identity associated with BMS transformations at past and future null infinity [11,12], Ref. [13] conjectured a tree-level next-to-soft generalisation of Eq. (2), where the subleading soft factor is given by

$$\mathcal{S}_{n,\mathrm{grav.}}^{(1)}=\sum_{i=1}^{n}\frac{\epsilon_{\mu\nu}(k)p_i^\mu k_\rho J^{(i)\rho\nu}}{p_i\cdot k}.\tag{3}$$

Here $J^{\rho\nu}$ is the total angular momentum associated with the hard external leg i, and Ref. [14] gave an analogous result for gauge theory:

$$\mathcal{S}_n^{(1)}=\sum_{i=1}^{n}\frac{\epsilon_\mu(k)k_\rho J^{(i)\mu\rho}}{p_i\cdot k}.\tag{4}$$

These results were subsequently understood from the point of view of the *scattering equations* of [15,16] in Ref. [17], using further symmetry arguments in [18,19], and string theoretic ideas in Refs. [20–22]. Higher dimensions were considered in Ref. [23], and a holographic description of the 4-dimensional gravitational theory pursued in [24]. Possible loop-level corrections to Eqs. (3), (4) have been examined in Refs. [25–27].

The aim of this paper is to explore the above results using Feynman diagrammatic methods previously developed in Refs. [7–9] (which are themselves related to the earlier results of Refs. [28,29]). Our main motivation is to clarify how those results are consistent with the recently proposed theorems. We stress that this analysis is new: whilst Refs. [7–9] and the much earlier work of Refs. [28,29] derive partial results regarding next-to-soft

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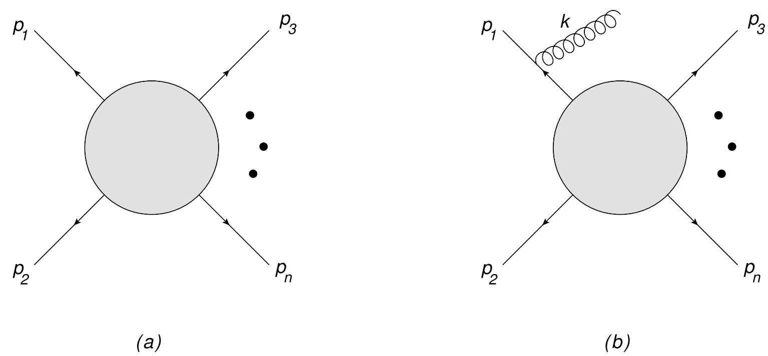

Fig. 1. (a) A hard interaction which produces n particles; (b) Emission of an eikonal gluon from an external leg.

corrections, they do not fully reproduce the results of Eqs. (3), (4). Secondly, it is nearly always useful to have multiple, equivalent ways of thinking about a given piece of physics, and we believe that our point of view may be useful in further studies of the next-to-soft theorems. Finally, connecting the recent results of Refs. [11–14,17–27] with Refs. [7–9] may aid the ongoing effort to use next-to-eikonal effects to increase the accuracy of collider predictions.

The structure of the paper is as follows. In Section 2 we briefly review the content of Refs. [7–9], addressing next-to-eikonal effects using effective Feynman rules. In Section 3, we show how these results reproduce the soft theorem of Eq. (4) for the case of scalar and fermionic emitting particles. The above references did not consider external gluons, and we perform this analysis in Section 4. We will confirm the tree-level results of Eq. (4) for external scalars, fermions and gluons. Finally, in Section 5 we discuss our results and conclude. Some technical details are presented in Appendix A.

2. Review of necessary concepts

In this section, we review the results of Refs. [7–9] and related papers, whose aim is to systematically classify next-to-eikonal contributions to scattering amplitudes in gauge and gravity theories. The starting point is to factorise the amplitude into a *hard function*, which is infrared finite, and a soft function, which collects all soft singularities.[1] Such a factorisation is well-known (see e.g. Ref. [30] for a review in QCD, and Refs. [31,32] for gravity). However, Refs. [7,9] generalised the soft function to include next-to-soft radiative corrections. The method proceeded by writing the propagators for the external particles in a background soft gauge field as first-quantised path integrals [33,34], which can be evaluated perturbatively. The leading term in this expansion is the eikonal approximation, in which external particles do not recoil, and change only by a (Wilson-line) phase [35]. The first subleading term describes the emission of next-to-soft gauge bosons, which are completely external to the hard interaction. Ref. [8] rederived the same results via a systematic expansion of Feynman diagrams to all orders in perturbation theory, and also checked the resulting formalism by reproducing known next-to-eikonal logarithms in Drell–Yan production. These are not the only sources of next-to-soft correction. As Refs. [7–9] explain in detail, one must also worry about soft gluon emissions which originate from inside the hard interaction.

[1] One must also include *jet functions* to keep track of collinear singularities. For the purposes of the present paper, however, we may implicitly absorb the jets into the hard function, as in Refs. [7–9].

Let us illustrate how the results apply to the present context, namely that of dressing an amplitude for the emission of n hard particles by an additional (next-to-)soft emission. One starts with a hard interaction such as that shown in Fig. 1(a). The leading soft singularities come from dressing all external legs (momenta $\{p_i\}$) with a soft gauge boson (momentum k), whose emission is described by an eikonal Feynman rule. This is shown in Fig. 1(b), and the kinematic parts of the eikonal Feynman rules for Yang–Mills theory and gravity are

$$\frac{p_i^\mu}{p_i\cdot k} \quad \text{and} \quad \frac{p_i^\mu p_i^\nu}{p\cdot k} \tag{5}$$

respectively. This clearly leads to the soft factors of Eqs. (3), (4), and at leading soft level one need only worry about the external emission of soft gluons. In Feynman diagram language, this can be understood by the fact that a soft gluon landing inside the hard interaction squares an offshell propagator, which dampens the infrared singular behaviour. In more physical terms, a soft gluon has an infinite Compton wavelength, and thus cannot resolve the substructure of the hard interaction. For the same reason, the above eikonal Feynman rules are independent of the spin of the hard emitting particles.

At next-to-soft level, there are two types of contribution. Firstly, there are next-to-soft gluon emissions external to the hard interaction, as shown in Fig. 2(a). These emissions are described by next-to-eikonal (NE) Feynman rules which, unlike the purely soft limit, depend on the spin of the emitting particles. External fermions and scalars were considered in Yang–Mills theory in Refs. [7,8]; scalars only were considered in the gravity study of Ref. [9]. In Yang–Mills theory, the NE Feynman rules for emission of a (potentially off-shell) gluon from a scalar and fermion are

$$V^\mu_{\text{scal.}} = \frac{k^\mu}{2p_i\cdot k} - \frac{k^2 p^\mu}{2(p_i\cdot k)^2}; \qquad V^\mu_{\text{ferm.}} = V^\mu_{\text{scal.}} - \frac{ik_\nu \Sigma^{\mu\nu}}{p_i\cdot k}, \tag{6}$$

where

$$\Sigma^{\mu\nu} = \frac{i}{4}\left[\gamma^\mu, \gamma^\nu\right] \tag{7}$$

is the generator of Lorentz transformations. There are also NE Feynman rules describing the correlated emission of a pair of soft gluons. The result for emission from an external scalar, for example, is

$$R^{\mu\nu} = \frac{p\cdot k_1 k_2^\mu p^\nu + p\cdot k_2 k_1^\nu p^\mu - p^\mu p^\nu k_1\cdot k_2 - \eta^{\mu\nu} p\cdot k_1 p\cdot k_2}{p\cdot k_1 p\cdot k_2 p\cdot(k_1+k_2)}, \tag{8}$$

where k_1^ν and k_2^μ are the soft gluon 4-momenta. An additional contribution arises for external fermions, again involving the generator