Physics Letters B 737 (2014) 379–382

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The missing asymptotic sector of rotating black-hole spectroscopy

Uri Keshet *, Arnon Ben-Meir

Physics Department, Ben-Gurion University of the Negev, PO Box 653, Be'er-Sheva 84105, Israel

ARTICLE INFO

Article history:
Received 16 August 2014
Accepted 5 September 2014
Available online 10 September 2014
Editor: M. Cvetič

ABSTRACT

The rotation of a Kerr black hole splits its low-frequency spectrum in two, so it was so far unclear why the known highly-damped resonances show no splitting. We find the missing, split sector, with spin s quasinormal modes approaching the total reflection frequencies $\omega(n \in \mathbb{N}) = -\Omega \Delta J - i\kappa(n-s)$, where Ω, κ and ΔJ are the horizon's angular velocity, surface gravity, and induced change in angular momentum. Surprisingly, the new sector is at least partly polar, and corresponds to reversible J transitions. Its fundamental branch converges quickly, possibly affecting gravitational wave signals. A simple interpretation of the Carter constant of motion is proposed.

1. Introduction

The natural vibrational modes of a black hole, known as quasi-normal modes (QNMs) due to their typically fast damping, play an important role in the classical study of black holes, in the search for a theory of quantum gravity, and in the gauge/gravity duality; see [1,2] for reviews.

The highly-damped modes, in particular, tend to show an interesting structure with simple or no dependence upon the vibration parameters. It was suggested that such an asymptotic QNM corresponds directly to a quantum transition, and for a Schwarzschild black hole may be interpreted as an elementary change in its surface area [3]. The subsequent flurry of black hole spectroscopy research produced many results, but for the most part, the asymptotic QNMs defied a simple interpretation along the lines of the correspondence principle [1].

Rotating, electrically neutral (Kerr) black holes are of special interest, due to their astronomical prevalence and as targets for gravitational wave searches. The asymptotic QNMs of rotating black holes were computed numerically [4] and analytically [5], but the resulting expressions for the frequencies are too complicated to admit a simple interpretation along the lines of [3]. However, when combined with the related total-reflection and total-transmission modes, these QNMs show an interesting structure, which provides hints at the microscopic description of the black hole [6].

Curiously, while the rotation is known to split the spectrum into two distinct branches of QNMs at low frequencies [7], only one asymptotic branch has been reported thus far, with no sign of splitting. These modes were identified as prolate, equatorial vibrations [6]. It is natural to ask what happens at asymptotic frequencies to the second QNM branch, and to polar excitations in general. The question is further motivated by some contradictory results in the literature, and because the very existence of a second asymptotic sector was largely overlooked.

We compute the Kerr QNMs up to high overtones, in order to uncover the fate of the elusive second branch. The transmission–reflection problem is then analyzed in the oblate limit, unexpectedly found to be relevant to the new sector. Some physical implications of the surprisingly simple form of the new QNMs are then discussed.

2. Formalism

Consider a Kerr black hole of mass M and angular momentum $J = aM$, with linear, massless field perturbations following Teukolsky's equation [8]. Decomposition into radial and angular wave functions, ${}_s\psi_{lm}(x) = e^{i(m\phi - \omega t)} {}_sS_{lm}(\mu \equiv \cos\theta) {}_sR_{lm}(r)$, leads to separate angular and radial equations linked by a coupling constant ${}_sA_{lm}$, which is closely related to the Carter constant [9], as we argue below. Here, $x = (t, r, \theta, \phi)$ are Boyer–Lindquist coordinates, and l, m are angular, azimuthal harmonic indices. The spin-weight parameter s gives the spin of the field, specializing the analysis to gravitational ($s = -2$), electromagnetic ($s = -1$), scalar ($s = 0$), or two-component fermion ($s = -1/2$) fields. We use Planck units where $G = c = k_B = \hbar = 1$, and shall henceforth omit the indices s, l, m when possible.

* Corresponding author.
E-mail address: ukeshet@bgu.ac.il (U. Keshet).

http://dx.doi.org/10.1016/j.physletb.2014.09.011
0370-2693/

Teukolsky's angular equation is [8]

$$\frac{d}{d\mu}\left[\left(1-\mu^2\right)\frac{dS}{d\mu}\right] = \left[\frac{(m+s\mu)^2}{1-\mu^2}-(a\omega\mu)^2+2a\omega s\mu-s-A\right]S. \quad (1)$$

The regularity of S at the poles $\mu=\pm1$ picks out a discrete set of solutions, known as spin-weighted spheroidal wave functions ([10], and references therein) and corresponding eigenvalues $A(\omega)$. For $s=0$, these functions reduce to the familiar spheroidal wave functions [11].

The radial equation can be written in the form [5]

$$\left(\frac{d^2}{dr^2}+\omega^2V^2\right)\left(\Delta^{\frac{s+1}{2}}R\right)=0, \quad (2)$$

where $\Delta\equiv r^2-2Mr+a^2$ vanishes at both horizons, $r_\pm\equiv M\pm(M^2-a^2)^{1/2}$. Here we defined $(\omega V)^2\equiv p_r^2+sp_s^2$,

$$p_r^2\equiv\frac{[(r^2+a^2)^2-a^2\Delta]\omega^2-4Mar\omega m-(\Delta-a^2)m^2-q\Delta}{\Delta^2}, \quad (3)$$

$(p_s\Delta)^2\equiv 2i[r\Delta-M(r^2-a^2)]\omega-(M-r)[2iam+s(M-r)]$, and $q\equiv A+s-m^2$. The QNMs are solutions of Eqs. (1) and (2) for physical boundary conditions of purely outgoing waves at both spatial infinity and the event horizon, i.e. crossing into the black hole. Equivalently, these solutions correspond to transmission and reflection by the black hole when the incident wave is negligible.

For given black hole (M,a) and field (s,l,m) parameters, the above constraints pick an infinite, discrete set of QNM solutions. These modes are labelled by an index $n\in\mathbb{N}$, and are specified by their complex frequency $\omega=\omega_R+i\omega_I$ and coupling constant A, where $\omega_I<0$ (time decay) diverges as $n\to\infty$. They show a complex–conjugate pairing symmetry, where for each $\{n,l,m\}$-QNM there is an $\{n,l,-m\}$-mode that satisfies [7]

$$\omega_{n,l,m}=-\omega^*_{n,l,-m} \quad \text{and} \quad A_{n,l,m}=A^*_{n,l,-m}. \quad (4)$$

We thus consider only $m\geq0$, so each mode is unique.

In addition to the pairing symmetry, the rotation of the black hole introduces a Zeeman-like splitting of the QNMs, such that for given parameters n, l and $m>0$ there are actually two modes, each belonging to a distinct QNM branch [7], which we label $+$ (new branch) and $-$ (known branch). Each such QNM pair coalesces into a single Schwarzschild mode as $a\to0$.

3. Series solution

For purely imaginary large ω, S is thought to be of the prolate-type, such that [11]

$$A^{(pr)}=iA_1a\omega+A_0+O\left(|a\omega|^{-1}\right), \quad (5)$$

where $A_1=2L+1$, $L\equiv l-\max(|m|,|s|)$ [10], and A_0 is a constant; $A_0=m^2-L(L+1)/2-3/4$ for $s=0$. It was previously argued that this applies to all QNMs in the highly-damped regime [4]. However, this assumption was found [4,5] to yield only one, equatorial [6] branch of QNMs, which therefore questions the validity of the assertion. To search for all QNMs, here we make no assumption regarding A, but rather solve both the angular equation (1) and the radial equation (2) simultaneously.

Low QNM frequencies are usually computed by series expansions of R and S, each leading to its own three-term recurrence relation. These commonly-used recursions were derived in [7] and will not be reproduced here; a simplified form of the radial recursion may be found in [12]. The QNMs are determined by requiring that both recursions simultaneously converge, leading to an infinite set of QNM parameter pairs $\{\omega_n, A_n\}$.

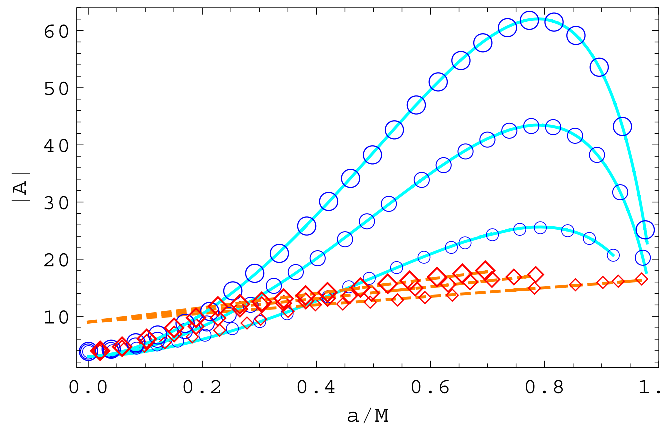

Fig. 1. Absolute value of the coupling constant A for fundamental modes in the new branch (circles) and in the known branch (diamonds), for $n=30$ (small symbols), 40 (medium), and 50 (large). The corresponding oblate (solid curves; Eq. (6)) and prolate (dashed curves; Eq. (5) with $A_0=9$) formulae fit well above an increasingly smaller threshold $a_s(n)$.

In the highly-damped regime, $n\gg1$, the series computation becomes increasingly challenging numerically. To find such QNMs, including both branches, we start with pairs of easily computed [13] Schwarzschild modes $\omega_n^\pm$, defined such that $\Re(\omega_n^+)=-\Re(\omega_n^-)>0$. Then we gradually increase a from 0, in sufficiently small steps to allow our root-finding algorithm to converge. The results are illustrated in Figs. 1 and 2.

Fig. 1 depicts A for gravitational modes with $-s=l=m=2$ (henceforth: fundamental modes). In the Schwarzschild limit, $A(a=0)=l(l+1)-s(s+1)$ for both branches [8]; A^+, A^- remain comparable at small values of the rotation parameter a. However, beyond a certain threshold a_s, the two branches show markedly different behaviors. While the known branch asymptotes to the familiar prolate scaling of Eq. (5), the new branch asymptotes to the oblate limit, given by [14]

$$A^{(ob)}=-(a\omega)^2+2qa\omega-\left(q^2-m^2+2s+1\right)+O\left(|a\omega|^{-1}\right), \quad (6)$$

where q was derived in [15], and in our case reduces to

$$q=\begin{cases}1-m+2(l+s) & \text{if } m>l+2s;\\ 1+l-\operatorname{mod}(l+m,2) & \text{otherwise.}\end{cases} \quad (7)$$

The split point a_s becomes gradually smaller at higher n; comparing Eqs. (5) and (6) suggests that $a_s\sim|A_1/\omega_I|$, such that $a_s\to0$ in the highly-damped limit.

Prolate and oblate behaviors are known to interchange in the complex ω plane, along branch cuts emanating from points where different eigenvalues A coalesce (*e.g.*, [16], and references therein). Our results suggest that such branch cuts are present near the imaginary axis for large $|\omega|$, at least for the fundamental mode (but see [10]). Note that both oblate and prolate asymptotic expansions are highly degenerate, $A_l^{(ob)}\simeq A_{l+1}^{(ob)}$ when $0\leq l-m+2s$ is even, and (at least when $s=0$) $A_m^{(pr)}\simeq A_{-m}^{(pr)}$.

Fig. 2 shows the real (figure body) and imaginary (inset) parts of the QNM frequencies of various modes. Consider first the new branch of fundamental modes, depicted as empty circles of size corresponding to n. It quickly converges to $\omega_R=m\Omega$ (solid curve), where $\Omega=4\pi a/\mathcal{A}$ and $\mathcal{A}=4\pi(r_+^2+a^2)$ are the angular velocity and surface area of the (outer) horizon. Non-fundamental $m>0$ modes do not converge as rapidly with n, but are seen to approach or oscillate around this value. All $m=0$ modes (filled symbols) rapidly converge to $\omega_R=0$.