Physics Letters B 737 (2014) 388–394

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Information-entropic measure of energy-degenerate kinks in two-field models

R.A.C. Correa [a,b,*], A. de Souza Dutra [a], M. Gleiser [b]

[a] UNESP – Campus de Guaratinguetá – DFQ, Av. Dr. Ariberto Pereira Cunha, 333 C.P. 205, 12516-410 Guaratinguetá, SP, Brazil
[b] Center for Cosmic Origins and Department of Physics and Astronomy, Dartmouth College, Hanover, NH 03755, USA

ARTICLE INFO

Article history:
Received 29 August 2014
Accepted 3 September 2014
Available online 16 September 2014
Editor: M. Trodden

Keywords:
Entropy
Nonlinear
Bloch walls
Kinks
Lumps

ABSTRACT

We investigate the existence and properties of kink-like solitons in a class of models with two interacting scalar fields. In particular, we focus on models that display both double and single-kink solutions, treatable analytically using the Bogomol'nyi–Prasad–Sommerfield bound (BPS). Such models are of interest in applications that include Skyrmions and various superstring-motivated theories. Exploring a region of parameter space where the energy for very different spatially-bound configurations is degenerate, we show that a newly-proposed momentum–space entropic measure called Configurational Entropy (CE) can distinguish between such energy-degenerate spatial profiles. This information-theoretic measure of spatial complexity provides a complementary perspective to situations where strictly energy-based arguments are inconclusive.

1. Introduction

Since the Scottish channel engineer John Scott Russell first discovered the existence of solitary waves in 1834 [1] and, in particular, since the 1960s and 70s [2–7], the study of nonlinear solutions of PDEs that preserve their spatial profile has attracted much interest in many areas of physics, such as in cosmology [8], field theory [9,10], condensed matter physics [11], and others [12]. In high-energy physics, solitons [10–13] are generally known as solutions of nonlinear field equations whose energy density is localized in space. Certain soliton solutions, as in the case of sine-Gordon kinks [13], have the interesting feature of keeping their shape unaltered after scattering with other solitons. (Here, we will use "soliton" to characterize solutions with localized energy-density, even if many may not maintain their spatial profile after scattering.)

Nowadays, the properties of nonlinear configurations are well understood in a wide class of models with or without spontaneous symmetry breaking, and with or without a nontrivial topological vacuum structure. Of particular interest to us here are kinks, non-dissipative solutions with an associated topological charge. Kink configurations arise in $(1+1)$-dimensional field theories when the scalar field potential has two or more degenerate minima. A well-known example is the ϕ^4-kink, also called the Z_2 kink [9,14]. In this case, a single real scalar field ϕ interpolates between the two degenerate minima of the potential.

A powerful insight to solve nonlinear problems analytically was introduced by Bogomol'nyi [15], Prasad and Sommerfield [16]. Known as the BPS bound, it is based on obtaining a first-order differential equation from the energy functional. With this method, it is possible to find solutions that minimize the energy of the configuration while ensuring their stability. A large variety of models in the literature use the BPS approach, such as solutions found in Skyrme models [17,18], monopoles [19,20], supersymmetric black holes [21], supergravity [22], and K-field theories [23].

A few decades ago, it was shown that it is possible to find kink-like solutions for certain coupled scalar field theories in $(1+1)$-dimensional models. Presented by Rajaraman, the approach is based on a "trial and error" method which leads to important particular solutions [24]. Bazeia and collaborators [25] showed that solutions of certain second-order differential systems with two or more scalar fields can be mapped into a corresponding set of first-order nonlinear differential equations, so that one can obtain the general solution of the system [26].

In an apparently disconnected topic, in 1948 Shannon defined the entropy of a data string as a measure of how much information is needed to characterize it in a transmission: the more information is needed for a reliable transmission, the higher the entropy.

* Corresponding author.
E-mail addresses: fis04132@gmail.com (R.A.C. Correa), dutra@gmail.com (A. de Souza Dutra), mgleiser@dartmouth.edu (M. Gleiser).

http://dx.doi.org/10.1016/j.physletb.2014.09.002

Inspired by Shannon, Gleiser and Stamatopoulos (GS) recently proposed a measure of complexity of a localized mathematical function [27]. GS proposed that the Fourier modes of square-integrable, bounded mathematical functions can be used to construct a measure of what they called configurational entropy (CE): a configuration consisting of a single mode has zero CE (a single wave in space), while one where all modes contribute with equal weight has maximal CE. To apply such ideas to physical models, GS used the energy density of a given spatially-localized field configuration, found from the solution – exact or approximate – of the related PDE. Of importance in what follows, GS pointed out that the configurational entropy can be used to choose the best-fitting trial function in situations where their energies are degenerate. More generally, the approach presented in [27] has been recently used to study the nonequilibrium dynamics of spontaneous symmetry breaking [28], to obtain a stability bound for compact astrophysical objects [29], and to investigate the emergence of localized objects during inflationary preheating [30].

In the present work we will compute the configurational entropy of some classes of models with two interacting scalar fields [24–26,31]. These models admit a variety of kink-like solutions, and have been shown to give rise to bags, junctions, and networks of BPS and non-BPS defects [32]. In particular, we will explore analytical solutions that are energy-degenerate but quite distinct in their spatial profiles. We will show that the CE can be used to distinguish between such configurations, adding a new information-theoretic perspective to the study of BPS solitons and other nonlinear localized configurations.

Section 2 introduces the model and its analytical solutions. Section 3 reviews the configurational entropy measure for spatially localized solutions. In Section 4 we compute the configurational entropy for two-field BPS solitons and show how it can be used to distinguish between energy-degenerate configurations. In Section 5 we present our conclusions and final remarks.

2. Interacting scalar field model and its solutions

Consider a $(1+1)$-dimensional model with two interacting real scalar fields described by the following Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\nu \phi)^2 + \frac{1}{2}(\partial_\nu \chi)^2 - V(\phi,\chi), \tag{1}$$

where $V(\phi,\chi)$ is the potential. We use units with $c=\hbar=1$ and metric $\eta_{\nu\beta}=\mathrm{diag}(1,-1)$ with coordinates $x^\nu=(t,x)$.

The potential $V(\phi,\chi)$ can be represented in terms of a superpotential $W(\phi,\chi)$ as

$$V(\phi,\chi) = \frac{1}{2}\left[\left(\frac{\partial W(\phi,\chi)}{\partial \phi}\right)^2 + \left(\frac{\partial W(\phi,\chi)}{\partial \chi}\right)^2\right]. \tag{2}$$

This representation includes supersymmetric models that generate distinct domain walls and topological solitons [33–35].

From the Lagrangian density (1) and the definition of the superpotential (2), the classical Euler–Lagrange equations of the static field configurations $\phi=\phi(x)$ and $\chi=\chi(x)$ are given by

$$\frac{d^2\phi}{dx^2} = W_\phi W_{\phi\phi} + W_\chi W_{\chi\phi}, \tag{3}$$

$$\frac{d^2\chi}{dx^2} = W_\chi W_{\chi\chi} + W_\phi W_{\chi\phi}, \tag{4}$$

where the subscripts denote derivatives with respect to the two fields. The energy functional of the static field configurations can be calculated as

$$E_{BPS} = \frac{1}{2}\int_{-\infty}^{\infty} dx\left[\left(\frac{d\phi}{dx}\right)^2 + \left(\frac{d\chi}{dx}\right)^2 + W_\phi^2 + W_\chi^2\right], \tag{5}$$

where $W_\phi \equiv \frac{\partial W(\phi,\chi)}{\partial\phi}$ and $W_\chi \equiv \frac{\partial W(\phi,\chi)}{\partial\chi}$. The above functional energy can be easily rewritten in the following form

$$E_{BPS} = \frac{1}{2}\int_{-\infty}^{\infty} dx\left[\left(\frac{d\phi}{dx} - W_\phi\right)^2 + \left(\frac{d\chi}{dx} - W_\chi\right)^2 + 2\left(W_\phi \frac{d\phi}{dx} + W_\chi \frac{d\chi}{dx}\right)\right]. \tag{6}$$

As a consequence, the solutions with minimal energy of the second-order differential equations for the static solutions can be found from the following two first-order equations

$$\frac{d\phi}{dx} = W_\phi, \quad \text{and} \quad \frac{d\chi}{dx} = W_\chi. \tag{7}$$

The energy E_{BPS}, which is called BPS energy, is written as

$$E_{BPS} = \left|W(\phi_j,\chi_j) - W(\phi_i,\chi_i)\right|, \tag{8}$$

where ϕ_i and χ_i denote the ith vacuum state of the model.

Following Ref. [26], it is possible from (7) to formally write the equation

$$\frac{d\phi}{W_\phi} = dx = \frac{d\chi}{W_\chi}, \tag{9}$$

which leads to

$$\frac{d\phi}{d\chi} = \frac{W_\phi}{W_\chi}. \tag{10}$$

The above equation is a nonlinear differential equation relating the scalar fields of the model so that $\phi=\phi(\chi)$. Once this function is known, Eqs. (7) become uncoupled and can be solved.

Considering the application below, we now review the model studied in Refs. [25,26,32], used for modeling a great number of systems [32–41], whose superpotential is given by

$$W(\phi,\chi) = -\lambda\phi + \frac{\lambda}{3}\phi^3 + \mu\phi\chi^2, \tag{11}$$

where λ and μ are real and positive dimensionless coupling constants. The potential $V(\phi,\chi)$ of the model with the above superpotential is given by

$$V(\phi,\chi) = \frac{1}{2}\left[\lambda^2 + \lambda^2\phi^2(\phi^2-2) + \mu^2\chi^2\left(\chi^2 - \frac{2\lambda}{\mu}\right) + 2\mu^2\left(\frac{\lambda}{\mu}+2\right)\phi^2\chi^2\right]. \tag{12}$$

For $\lambda/\mu > 0$ the model has four supersymmetric minima (ϕ,χ)

$$\mathcal{M}_1 = (-1,0), \qquad \mathcal{M}_2 = (1,0),$$

$$\mathcal{M}_3 = \left(0,-\sqrt{\frac{\lambda}{\mu}}\right), \qquad \mathcal{M}_4 = \left(0,\sqrt{\frac{\lambda}{\mu}}\right). \tag{13}$$

The orbits connecting the vacuum states can be seen on Fig. 1. Note that we can have six configurations connecting the vacua, where five are BPS states and one is non-BPS.

Using the above results, the sectors connecting the vacua and their corresponding energies are given by