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# Finite temperature Casimir effect in spacetime with extra compactified dimensions

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ABSTRACT

In this Letter, we derive the explicit exact formulas for the finite temperature Casimir force acting on a pair of parallel plates in the presence of extra compactified dimensions within the framework of Kaluza–Klein theory. Using the piston analysis, we show that at any temperature, the Casimir force due to massless scalar field with Dirichlet boundary conditions on the plates is always attractive and the effect of extra dimensions becomes stronger when the size or number of the extra dimensions increases. These properties are not affected by the explicit geometry and topology of the Kaluza–Klein space.


Theories with extra space dimensions can be dated back to the work of Kaluza and Klein [1,2] in an attempt to unify gravity and classical electrodynamics. The advent of string theory has made the idea of extra dimensions indispensable [3]. Recently, the interest for physical models that contain extra warped dimensions come from the endeavor to explain the large gap between the Planck and electroweak scale (the hierarchy problem), and the dark energy and cosmological constant problem [4–25]. In this Letter, we are interested in studying the Casimir effect in the presence of extra compactified dimensions. The research on Casimir effect in the scenarios with extra dimensions have been studied in the context of string theory [26–29], dark energy and cosmological constant [16–19,21,22,25,30–37], as well as stabilization of extra dimensions [23,24,38–53]. Casimir force for massless scalar field with Dirichlet boundary conditions on a pair of parallel plates inside space with extra dimensions compactified to a torus $T^n = (S^1)^n$ have been calculated in [54–59]. The dependence of the Casimir energy on a cut-off scale was investigated in [25]. Casimir effect for electromagnetic fields confined between a pair of parallel and perfectly conducting plates in $\mathbb{R}^{d+1}$ was considered in [60]. In the case the extra dimension is compactified to a circle $S^1$, it was studied in [25,61–63]. For the Randall–Sundrum spacetime model, the Casimir force due to massless scalar fields subject to Dirichlet boundary conditions on two parallel plates were calculated in [64–66].

Despite the importance of the thermal corrections, most of the works mentioned above dealt with Casimir effect at zero temperature. In particular, we would like to mention the two papers by Cheng [56,58] where he calculated the Casimir force acting on a pair of parallel plates due to massless scalar field with Dirichlet boundary conditions in spacetime with $n$ extra dimensions compactified to a torus. As pointed out in the comment [59], the earlier paper [56] concluded that the presence of extra dimensions will give rise to repulsive Casimir force under certain conditions; but this result was invalidated by the author himself in the later paper [58], where he redid the calculations in a more physical setup known as piston [67] and concluded that the Casimir force shall always be attractive. The finite temperature Casimir energy in $(d+1)$-dimensional rectangular cavities were first calculated in [68] and revisited in [69]. In [70], we calculated the finite temperature Casimir force on a $(d+1)$-dimensional piston inside a rectangular cavity due to massless scalar field as well as electromagnetic field and concluded that the Casimir force is always attractive when one end of the cavity was opened. As a result, it will be natural to suspect the validity of the result of Cheng [55], who concluded the possible repulsive Casimir force at finite temperature for a pair of parallel plates in space with an extra dimension compactified to a circle.

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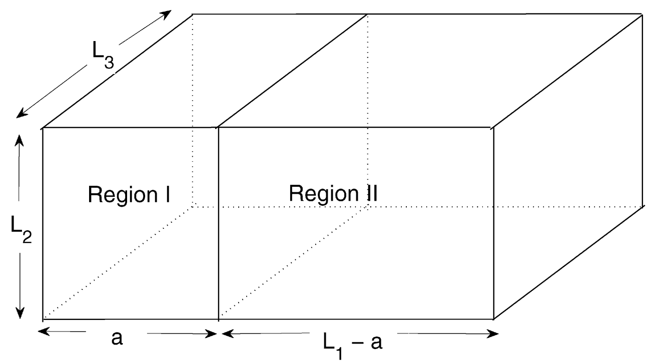


**Fig. 1.** A three-dimensional rectangular piston.

In this Letter, we consider the Casimir force at finite temperature within the piston setup in spacetime with $n$ extra dimensions compactified to a torus $T^n = (S^1)^n$. More precisely, we consider a piston inside a three-dimensional rectangular cavity (see Fig. 1) in a space with $n$-extra dimensions and calculated the Casimir force acting on the piston for a massless scalar field with Dirichlet boundary conditions on the walls of the rectangular cavity and the piston. At the end, we let $L_1$, $L_2$, $L_3$ approach infinity so that we obtain finite temperature Casimir force acting on parallel plates in the presence of extra compactified dimensions. We are also going to comment on the case where the extra dimensions are compactified to an arbitrary compact manifold without boundary.

For a massless scalar field in the spacetime $M^4 \times T^n$, where $M^4$ is the $(3+1)$-dimensional Minkowski spacetime, a complete set of eigenfrequencies of the field confined within a rectangular cavity $[0, L_1] \times [0, L_2] \times [0, L_3]$ subject to the Dirichlet boundary conditions are given by

$$\omega_{\boldsymbol{k},\boldsymbol{l}} = c\sqrt{\left(\frac{\pi k_1}{L_1}\right)^2 + \left(\frac{\pi k_2}{L_2}\right)^2 + \left(\frac{\pi k_3}{L_3}\right)^2 + \sum_{j=1}^{n}\left(\frac{l_j}{R_j}\right)^2},$$

where $c$ is the light speed, $\boldsymbol{k} = (k_1, k_2, k_3) \in \mathbb{N}^3$ and $\boldsymbol{l} = (l_1, \ldots, l_n) \in \mathbb{Z}^n$. Notice that in contrast to [54–58], our $l_j$, $j = 1, \ldots, n$, runs from $-\infty$ to $\infty$ instead of 0 to $\infty$. As pointed out in [59], this is more natural since the extra dimensions are compactified to a torus, a manifold without boundary. Therefore for each compact dimension, we need to consider a complete set of eigenvalues for Laplace operators on $S^1$ counting with multiplicities.

The finite temperature Casimir energy due to the vacuum fluctuation of the field inside the rectangular cavity is defined by the mode sum:

$$E_{\text{Cas}}^{\text{cavity}}(T) = \frac{\hbar}{2}\sum \omega + k_B T \sum \log\left(1 - e^{-\hbar\omega/k_B T}\right),$$

where the summations run through all eigenfrequencies $\omega$, $T$ is the temperature at equilibrium, $\hbar$ is the reduced Planck constant and $k_B$ is the Boltzmann constant. For regularization purpose, we introduce a cut-off $\lambda$ so that

$$E_{\text{Cas}}^{\text{cavity}}(\lambda; T) = \frac{\hbar}{2}\sum \omega e^{-\lambda\omega} + k_B T \sum \log\left(1 - e^{-\hbar\omega/k_B T}\right).$$

As pointed out in [67], the Casimir energy of the piston system (Fig. 1) is the sum

$$E_{\text{Cas}}(\lambda; a; T) = E_{\text{Cas}}^{\text{cavity}}(\lambda; T)\big|_{L_1 \to a} + E_{\text{Cas}}^{\text{cavity}}(\lambda; T)\big|_{L_1 \to L_1 - a} + E_{\text{Cas}}^{\text{ext}}(T)$$

of the Casimir energies of Region I, Region II and the exterior region. Being independent of the position of the piston, the Casimir energy of the exterior region do not contribute to the Casimir force acting on the piston. As a result, the Casimir force acting on the piston is given by

$$F_{\text{Cas}}(a; T) = -\lim_{\lambda \to 0^+} \frac{\partial}{\partial a}\left\{E_{\text{Cas}}^{\text{cavity}}(\lambda; T)\big|_{L_1 \to a} + E_{\text{Cas}}^{\text{cavity}}(\lambda; T)\big|_{L_1 \to L_1 - a}\right\}.$$

A distinction about the piston setup is that the Casimir force $F_{\text{Cas}}(a; T)$ is finite in the $\lambda \to 0^+$ limit. Using the same method of computation as presented in [70], we find that in the $L_1 \to \infty$ limit,

$$F_{\text{Cas}}(a; T) = -\pi k_B T \sum_{(k_2,k_3) \in \mathbb{N}^2} \sum_{\boldsymbol{l} \in \mathbb{Z}^n} \sum_{p=-\infty}^{\infty} \frac{\Lambda_{k_2,k_3,\boldsymbol{l},p}}{\exp(2\pi a \Lambda_{k_2,k_3,\boldsymbol{l},p}) - 1} \tag{1}$$

where

$$\Lambda_{k_2,k_3,\boldsymbol{l},p} = \sqrt{\left(\frac{k_2}{L_2}\right)^2 + \left(\frac{k_3}{L_3}\right)^2 + \frac{1}{\pi^2}\sum_{j=1}^{n}\left(\frac{l_j}{R_j}\right)^2 + \left(\frac{2pk_B T}{\hbar c}\right)^2}.$$

Eq. (1) shows clearly that for an opened piston, the Casimir force is always attractive at *any temperature* and the magnitude of the Casimir force is a decreasing function of the distance $a$ between the piston and the opposite wall. Moreover, since each individual term in the summation of (1) is positive, it shows that the magnitude of the Casimir force becomes larger as the number of extra dimensions