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A gracious exit from the matter-dominated phase in a quantum cosmic phantom model

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ABSTRACT

The most recent observational constraints coming from Planck, when combined with other cosmological data, provide evidence for a phantom scenario. In this work we consider a quantum cosmic phantom model where both the matter particles and scalar field are associated with quantum potentials which make the effective mass associated with the matter particles to vanish at the time of matter-radiation equality, resulting in a cosmic system where a matter dominance phase followed by an accelerating expansion is allowed.

The problem of dark energy still remains unsolved. Its equation of state (EoS), which is defined as $w = p/\rho$, where p and ρ are the pressure and energy density of dark energy, respectively, could be in the phantom regime ($w < -1$) [1] according to the most recent observational constraints [2]. Planck latest results [2] plus WMAP low-l polarisation (WP), when combined with Supernova Legacy Survey (SNLS) data, favour the phantom domain at 2σ level for a constant w

$$w = -1.13^{+0.13}_{-0.14}\ (95\%;\ \mathit{Planck} + \mathit{WP} + \mathit{SNLS}), \tag{1}$$

while the Union2.1 compilation of 580 Type Ia supernovae (SNe Ia) is more consistent with a cosmological constant ($w = -1$). If we combine Planck + WP with measurements of H_0 [3], we get for a constant w

$$w = -1.24^{+0.18}_{-0.19} \tag{2}$$

which is in tension with $w = -1$ at more than the 2σ level. The constant w models are of limited physical interest. If $w \neq -1$ then it is likely to change with time. For a flat universe and for a non-constant w ($w = w_0 + w_a(1 - a)$ [4,5]) the combined data from Planck + WP + H_0 leads to

$$w_0 = -1.04^{+0.72}_{-0.69} \tag{3}$$

with a negative w_a, away from $w = -1$ at just under the 2σ level. Furthermore, with the release of the first results from Planck [2], claims for $w < -1$ at $\geq 2\sigma$ have been presented, such as [6], which features high-quality data and a careful analysis including systematic errors [7]. Also, the authors in [8] found that for the SNLS3 and the Pan-STARRS1 survey (PS1 SN) data sets, the combined SNe Ia + Baryon Acoustic Oscillations (BAO) + Planck data yield a phantom equation of state at $\sim 1.9\sigma$ confidence. Therefore, we find ourselves in a situation in which we can say [8], at 2σ confidence level, that given Planck data, either the SNLS3 and PS1 data have systematics that have not been accounted for yet, or the Hubble constant is below 71 km/s/Mpc, or else $w < -1$.

The above observational results, in addition to theoretical motivations, are compelling enough to justify the study of the phantom regime in more depth. Given that the standard cosmological model (ΛCDM) with $w = -1$ cannot accommodate this scenario, different solutions have been proposed. There are two main approaches. The first one includes a scalar field with a negative kinetic energy term [1] but this leads to violent quantum instabilities [9,10]. The second one is more radical and advocates a modification of general relativity. In this modified gravity scenario there are prescriptions that do not have any ghost degree of freedom, such as the Brans–Dicke type gravity [11], the scalar-Einstein–Gauss–Bonnet gravity [12], and the $F(R)$ gravity [13]. These three proposals are also free of perturbative instabilities but one should also investigate the corrections to the Newton law, perform the PPN analysis [14] etc., in order to ensure that they are consistent with the more accurate solar-system and experimental data. Furthermore, it was recently realised by some authors that the most general second order scalar tensor Lagrangian (and thus, ghost-free) that still produces second order equations of motion is the

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so-called Horndeski Lagrangian [15–18], a model that includes four arbitrary functions of the scalar field and its kinetic energy, and of which Brans–Dicke, Gauss–Bonnet and $F(R)$ are just particular examples.

Alternatively, a theory which is self-consistent and agrees with all the above observational data [2] has been proposed [19–21]. It is most economical as it only uses general relativity and quantum mechanics without inserting any kind of vacuum fields or introducing any extra terms in the Hilbert–Einstein gravitational action. In such a framework one can get essentially two relevant quantum solutions both of which can be seen as quantum perturbations to the de Sitter space [20], which is recovered in the classical limit where $\hbar \to 0$. It has also been shown that out of these two possible solutions only one of them satisfies the second law of thermodynamics [21], and hence is physically meaningful. It corresponds to a phantom universe [1] but does not show any quantum instability [9,10] nor the sort of inconsistency coming from having a negative kinetic term for the scalar field – in fact, these models do not actually contain any scalar or other kinds of vacuum fields in their final equations and do not show neither a future singularity (Big Rip) [1,22] nor classical violations of the energy conditions. It is for these reasons that such a cosmic model has also been denoted as [20,21] *benigner* phantom model.

On the other hand, in Ref. [23] (see also [24]) it was shown that it is impossible to find a sequence of matter and scaling acceleration for any scaling Lagrangian which can be approximated as a polynomial because a scaling Lagrangian is always singular in the phase space so that either the matter-dominated era is prevented or the region with a viable matter is isolated from that where the scaling acceleration occurs. Such as it happens with other aspects of the current accelerating cosmology, the problem is to some extend reminiscent of the difficulty initially confronted by earliest inflationary accelerating models [25] which could not smoothly connect with the following Friedmann–Robertson–Walker (FRW) decelerating evolution [26]. As is well known, such a difficulty was solved by invoking the new inflationary scenario [27]. In fact, the problem posed in [23] for dark energy can be formulated by saying that a previous decelerating matter-dominated era cannot be followed by an accelerating universe dominated by dark energy and it is in this sense that it can be somehow regarded as the time-reversed version of the early inflationary exit difficulty. Ways out from this problem required assuming either a sudden emergence of dark energy domination or a cyclic occurrence of dark energy, both assumptions being quite hard to explain and implement. The aim of this work is to show that in the *benigner* phantom model [20,21] such problems are no longer present due to the quantum characteristics that can be assigned to particles and radiation in this model.

If we apply the real part of the Klein–Gordon wave equation to a quasi-classical wave function $R\exp(iS/\hbar)$, where the probability amplitude R ($P = |R|^2$) and the action S are real functions of the relativistic coordinates, and define the classical energy $E = \partial S/\partial t$ and momentum $p = \nabla S$, we can write the modified Hamilton–Jacobi equation

$$E^2 - p^2 + \tilde{V}_Q^2 = m_0^2, \tag{4}$$

where m_0 is the rest mass of the involved particle and $\tilde{V}_Q$ is a relativistic quantum potential,

$$\tilde{V}_Q^2 = \frac{\hbar^2}{R}\left(\nabla^2 R - \frac{\partial^2 R}{\partial t^2}\right), \tag{5}$$

which should be interpreted according to Bohm's idea [30] as the hidden quantum potential that accounts for precisely defined unobservable relativistic variables whose effects would physically manifest in terms of the indeterministic behaviour shown by the given particles. From Eq. (4) it immediately follows that $p = \sqrt{E^2 + \tilde{V}_Q^2 - m_0^2}$. Thus, since classically $p = \partial\tilde{L}/\partial[\dot{q(t)}]$ (with $\tilde{L}$ being the Lagrangian of the system and q the spatial coordinates, which depends only on time t, $q \equiv q(t)$), we have for the Lagrangian

$$\tilde{L} = \int d\dot{q}p = \int dv\sqrt{\frac{m_0^2}{1-v^2} + M^2}, \tag{6}$$

in which $v = \dot{q}$ and $M^2 = \tilde{V}_Q^2 - m_0^2$. In the classical limit $\hbar \to 0$, $\tilde{V}_Q \to 0$, and hence we are just left with the classical relativistic Lagrangian for a particle with rest mass m_0.

We start with an action integral that contains all the ingredients of our model. Such an action is a generalisation of the one used in [23] which contains a time-dependent coupling between dark energy and matter and leads to a general Lagrangian that admits scaling solutions formally the same as those derived in [23]. Setting the Planck mass to unity, our Lorentzian action reads

$$S = \int d^4x\sqrt{-g}\left[R + p(X,\phi)\right] + S_m\left[\psi_i, \xi, m_i(\tilde{V}_Q), \phi, g_{\mu\nu}\right] + ST(K, \psi_i, \xi), \tag{7}$$

where g is the determinant of the four-metric, p is a generically non-canonical general Lagrangian for the dark energy scalar field ϕ with kinetic term $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, formally the same as the one used in [23], S_m corresponds to the Lagrangian for the matter fields ψ_i, each with mass m_i, which is going to depend on the quantum potential $\tilde{V}_Q$ in a way that will be made clear in what follows, so as on the time-dependent coupling ξ of the matter field to the dark energy field ϕ. The term ST denotes the surface term which generally depends on the trace on the second fundamental form K, the matter fields ψ_i and the time-dependent coupling $\xi(t)$ between ψ_i and ϕ for the following reasons.

We first of all point out that in the theory being considered the coupling between the matter and the scalar fields can generally be regarded to be equivalent to a coupling between the matter fields and gravity plus a set of potential energy terms for the matter fields. In fact, if we restrict ourselves to this kind of theories, a scalar field ϕ can always be mathematically expressed in terms of the scalar curvature R [28]. More precisely, for the scaling accelerating phase we shall consider a quantum dark energy model (see [30,20,21]) in which the Lagrangian for the field ϕ vanishes in the classical limit where the quantum potential is made zero; i.e. we take $p = L = -V(\phi)(E(x,k) - \sqrt{1-\dot{\phi}^2})$, where $V(\phi)$ is the density of potential energy associated to the field ϕ and $E(x,k)$ is the elliptic integral of the second kind, with $x = \arcsin\sqrt{1-\dot{\phi}^2}$ and $k = \sqrt{1 - V_Q^2/V(\phi)^2}$, and the overhead dot $\dot{}$ means derivative with respect to time. We do not expect $\tilde{V}_Q$ to remain constant along the universal expansion but to increase like the volume of the universe $\mathrm{V} \propto a^3$ does. It is the quantum potential density $V_Q = \tilde{V}_Q/\mathrm{V}$ appearing in the Lagrangian L what should be expected to remain constant at all cosmic times. Using then a potential energy density for ϕ and the quantum medium [note that the quantum potential energy density becomes constant [20,21] (see later on)], we have for the energy density and pressure, $\rho \propto X(HV_Q/\dot{H})^2 = p(X)/w(t)$, with $H \propto \phi V_Q + H_0$, $\dot{H} \propto \sqrt{2X}V_Q$, where H_0 is constant. For the resulting field theory to be finite, the condition that $2X = 1$ (i.e. $\phi = C_1 + t$) had to be satisfied [20,21], and from the Friedmann equation the scale factor ought to be given by $a(t) \propto \exp(C_2 t + C_3 t^2)$, with C_1, C_2 and C_3 being constants. It follows then that for at least a flat space–time, we generally have $R \propto 1 + \alpha\phi^2$ (where α is another constant and we have