Physics Letters B 733 (2014) 151–157

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Pion cloud effects on baryon masses

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ARTICLE INFO

Article history:
Received 21 January 2014
Received in revised form 14 March 2014
Accepted 14 April 2014
Available online 18 April 2014
Editor: A. Ringwald

Keywords:
Pion cloud effects
Nucleon sigma term
Faddeev equations

ABSTRACT

In this work we explore the effect of pion cloud contributions to the mass of the nucleon and the Δ baryon. To this end we solve a coupled system of Dyson–Schwinger equations for the quark propagator, a Bethe–Salpeter equation for the pion and a three-body Faddeev equation for the baryons. In the quark–gluon interaction we explicitly resolve the term responsible for the back-coupling of the pion onto the quark, representing rainbow-ladder like pion cloud effects in bound states. We study the dependence of the resulting baryon masses on the current quark mass and discuss the internal structure of the baryons in terms of a partial wave decomposition. We furthermore determine values for the nucleon and Δ sigma-terms.

1. Introduction

The application of continuum functional methods to hadron physics phenomenology aims at the calculation of hadronic properties using the elementary degrees of freedom of Quantum Chromodynamics (QCD). In this framework mesons and baryons are considered as bound states of quarks and, hence, described by two-body Bethe–Salpeter equations (BSEs) and three-body Faddeev equations. These equations rely upon the knowledge of several QCD's Green's functions which are in turn solutions of Dyson–Schwinger equations (DSEs). The approach has the advantage that the origin of physical observables can be understood from the microscopic dynamics of quarks and gluons. Moreover, it is Poincaré covariant and is applicable at any momentum range.

As is well known, however, it is impossible to carry out this program exactly and truncations of both the DSEs and the bound state equations must be defined. The simplest one consistent with Poincaré covariance as well as constraints from chiral symmetry is the rainbow-ladder truncation (RL). Approximations of this kind have been extensively used in hadron calculations (see e.g. [1,2] for overviews) and turn out to be rather successful in reproducing, e.g., ground-state masses in selected channels.

There are, however, also severe limitations to the rainbow-ladder scheme. Consequently, much work has been invested in the past years on its extension towards more advanced approximations of the quark–gluon interaction. On the one hand, this may be accomplished directly by devising improved *ansätze* for the dressing functions of the quark–gluon vertex [3–6]. On the other hand, it is promising to work with diagrammatic approximations to the vertex DSE. While most studies so far concentrated on ($1/N_c$-subleading) Abelian contributions to the vertex (see e.g. [7–11]), the impact of the $1/N_c$-leading, non-Abelian diagram on light meson masses has been investigated in [12]. In addition, important unquenching effects in the quark–gluon interaction may be approximated by the inclusion of hadronic degrees of freedom [13–15]. This is possible, since the vertex DSE can be decomposed on a diagrammatic level into terms that are already present in the quenched theory and those involving explicit quark-loops. The latter ones can be expressed involving hadronic degrees of freedom. To leading order in the hadron masses, pion exchange between quarks is dominating these contributions. These pions are not elementary fields. Consequently, their wave functions need to be determined from their Bethe–Salpeter equation.

Having explicit hadronic degrees of freedom in the system may also be very beneficial for phenomenological applications of the approach. Pion cloud effects are expected to play an important role in the low momentum behavior of form factors and hadronic decay processes of baryons [16–23]. Within the covariant BSE-approach, the influence of pion back-coupling effects in the mass and decay constants of the pion itself and other light mesons has been studied in [15]. In the present work, we take this framework one step further and extend it to the covariant three-body calculations of nucleon and delta masses [24–26].

This letter is organized as follows: in Section 2 we review the main elements of the DSE/BSE framework and define the truncations and model used in this work. We present and discuss the

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http://dx.doi.org/10.1016/j.physletb.2014.04.031

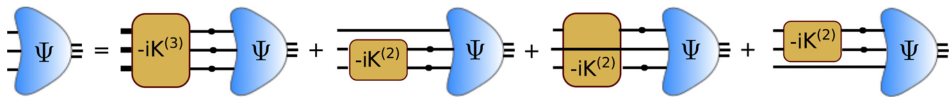

Fig. 1. Diagrammatic representation of the three-body Bethe–Salpeter equation.

results of our calculations in Section 3. Finally, some concluding remarks are made in Section 4.

2. Covariant three-body equation

The mass and internal structure of baryons are given, in a covariant Faddeev approach, by the solutions of the three-body equation (see Fig. 1)

$$\Psi = -i\widetilde{K}^{(3)}\, G_0^{(3)}\, \Psi + \sum_{a=1}^{3} -i\widetilde{K}^{(2)}_{(a)}\, G_0^{(3)}\, \Psi, \tag{1}$$

where $\widetilde{K}^{(3)}$ and $\widetilde{K}^{(2)}$ are the three- and two-body interaction kernels, respectively, and G_0 represents the product of three fully-dressed quark propagators S. We used here a compact notation where indices have been omitted and we assume that discrete and continuous variables are summed or integrated over, respectively. The spin-momentum part of the full amplitude Ψ depends on the total and two relative momenta of the three valence quarks inside the baryon. As discussed in more detail in Section 3.2, this amplitude contains all possible spin and orbital angular momentum contributions.

The quark propagators are obtained from their respective DSE

$$S^{-1}(p) = S_0^{-1}(p) - Z_{1f}\int_q \Gamma^{\nu}_{gqq,0} D_{\mu\nu}(p-q)\Gamma^{\nu}_{gqq}(p,q)S(q), \tag{2}$$

where the integration over the four-momentum q is abbreviated by $\int_q \equiv \int d^4q/(2\pi)^4$, S_0 is the (renormalized) bare propagator with its inverse given by

$$S_0^{-1}(p) = Z_2(i\not{p} + m_q), \tag{3}$$

with bare quark mass m_q, whereas

$$S^{-1}(p) = i\not{p}A\big(p^2\big) + B\big(p^2\big), \tag{4}$$

denotes the inverse dressed propagator. The renormalization point invariant running quark mass $M(p^2)$ is defined by the ratio of the scalar quark dressing function $B(p^2)$ and the vector dressing function $A(p^2)$: $M(p^2) = B(p^2)/A(p^2)$. Γ^{ν}_{gqq} is the full quark–gluon vertex with its bare counterpart $\Gamma^{\nu}_{gqq,0}$, $D^{\mu\nu}$ is the full gluon propagator and Z_{1f} and Z_2 are renormalization constants.

To solve the system formed by Eqs. (1) and (2) one needs to know the interaction kernels and the full quark–gluon vertex. The latter could in principle be obtained from the infinite system of coupled DSEs of QCD. In practice, however, this system has to be truncated into something manageable, which implies that educated *ansätze* have to be used for the Green's functions one is not solving for. The interaction kernels, in contrast, do not appear directly in the system of QCD's DSEs. In the quark–antiquark channel, a connection of those with the quark–gluon interaction is established via the axial-vector Ward–Takahashi identity, which ensures the correct implementation of chiral symmetry in the bound state equations [27,28]. In turn, it is natural from a systematic point of view to treat the interaction kernels in the quark–quark channels on a similar approximation level, such that both kernels are fixed once the approximation of the quark–gluon interaction is specified. This will be detailed below.

2.1. Rainbow-ladder truncation

The simplest and most commonly used ansatz for the quark–gluon and quark–quark interactions is the rainbow-ladder (RL) truncation. Here, only the tree-level flavor, color and Lorentz structures are kept for the quark–gluon vertex, so that the quark DSE reads

$$S^{-1}_{\alpha\beta}(p) = S^{-1}_{0,\alpha\beta}(p) - \int_q \widetilde{K}^{RL}_{\alpha\alpha'\beta'\beta}(k) S_{\alpha'\beta'}(q), \tag{5}$$

with momentum $k = p - q$ and kernel

$$\widetilde{K}^{RL}_{\alpha\alpha'\beta'\beta}(k) = -4\pi C\, Z_2^2 \frac{\alpha_{\text{eff}}(k^2)}{k^2} T_{\mu\nu}(k)\, \gamma^{\mu}_{\alpha\alpha'}\gamma^{\nu}_{\beta'\beta}. \tag{6}$$

Here Z_2 denotes the quark renormalization constant, $T_{\mu\nu}(k)$ the transverse projector

$$T_{\mu\nu}(k) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \tag{7}$$

and $C = 4/3$ the resulting color factor for quarks in fundamental representation. The effective coupling α_{eff} combines the non-perturbative dressing of the gluon propagator and the γ_μ-structure of the vertex. At large momenta, it is constrained by perturbation theory, whereas at low momenta we have to supply a model. In this work we use the model proposed in [29,30]

$$\alpha_{\text{eff}}\big(q^2\big) = \pi\eta^7\left(\frac{q^2}{\Lambda^2}\right)^2 e^{-\eta^2\frac{q^2}{\Lambda^2}} + \frac{2\pi\gamma_m\big(1 - e^{-q^2/\Lambda_t^2}\big)}{\ln[e^2 - 1 + (1 + q^2/\Lambda^2_{QCD})^2]}, \tag{8}$$

where for the anomalous dimension we use $\gamma_m = 12/(11N_C - 2N_f) = 12/25$, corresponding to $N_f = 4$ flavors and $N_c = 3$ colors. We fix the QCD scale to $\Lambda_{QCD} = 0.234$ GeV and the scale $\Lambda_t = 1$ GeV is introduced for technical reasons and has no impact on the results. The interaction strength is characterized by an energy scale Λ and the dimensionless parameter η controls the width of the interaction. They have to be fixed by experimental input, see Section 3.

The quark–antiquark kernel in the pion Bethe–Salpeter equation (BSE) has to match the interaction model in the quark-DSE such as to guarantee the Goldstone-boson property of the pion in the chiral limit. This is encoded in the axial-vector Ward–Takahashi identity (axWTI). In the rainbow-ladder truncation, the quark–antiquark kernel in the BSE is then also given by Eq. (6). The corresponding kernel describing the interaction between two quarks can be obtained via crossing symmetry. For our rainbow-ladder scheme this results in the same expression Eq. (6) with modified color factor $C = -2/3$. For diquarks, such a kernel together with its extensions has been explored e.g. in [7], whereas in the context of the three-body Faddeev equations first results have been reported in [24–26]. In the latter studies, the three-body irreducible interactions between the three quarks have been neglected. We adopt the same framework in this work.