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Singularity-free gravitational collapse and asymptotic safety

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ABSTRACT

A general class of quantum improved stellar models with interiors composed of non-interacting (*dust*) particles is obtained and analyzed in a framework compatible with asymptotic safety. First, the effective exterior, based on the Quantum Einstein Gravity approach to asymptotic safety is presented and, second, its effective compatible dust interiors are deduced. The resulting stellar models appear to be devoid of shell-focusing singularities.

1. Introduction

Some time ago, it was suggested by Steven Weinberg [1] that a quantum theory of gravitation may dynamically evade the divergences found in perturbative gravity. Specifically, this scenario, called *asymptotic safety*, implies the UV completion of gravity based on a non-Gaussian fixed point of the Renormalization Group flow. At the present time and thanks to the advent of new functional renormalization group methods, there is accumulating evidence in favor of the asymptotic safety scenario (see [2,3] and references therein); however, there are still some aspects of the approach that need clarification.

One of these aspects is that, since it seems only natural to demand that a truly fundamental theory of quantum gravity should be devoid of singularities, asymptotic safety should be able to provide singularity-free solutions as the result, for example, of a gravitational collapse. However, it is still a mystery whether and how the singularities that appeared in General Relativity (GR) would be avoided in the asymptotic safety scenario. The difficulty relies on the complexity of a full approach to the collapse of matter in the framework of asymptotic safety. In fact, to my knowledge, there has only been one previous approximation to this problem [4] which suggests that the deviations from GR offered by the asymptotic safety approach could be too small to prevent the generation of singularities during gravitational collapse.

This Letter aims to contribute to the analysis of the presence/absence of singularities in the framework of asymptotic safety. In particular, an attempt will be made to obtain and analyze the general class of stellar models consisting of non-interacting particles which are compatible with asymptotic safety. It is worth recalling that, in the framework of GR, the class of spherically symmetric solutions consisting of non-interacting (or *dust*) particles are known as the Lemaitre–Tolman–Bondi (LTB) solutions and have been thoroughly studied (see, for example, [5] and references therein). Since there is nothing preventing the collapse in these classical models, once the particles start collapsing they will be eventually forced to generate a singularity. From this classical point of view, the only question is whether this singularity will be space-like and hidden from any observers (as in the Oppenheimer–Snyder model [6]) or it will form a naked singularity visible to, at least, some observers. In fact, it has been shown [7,8] that the class of the LTB models is wide enough to admit both hidden and (locally or globally) naked singularities. The final goal of this Letter is to show that the dust models compatible with asymptotic safety, unlike their analogous classical LTB models, are singularity-free.

The Letter has been divided as follows. In Section 2 the improved stellar exterior coming from the asymptotic safe approach is presented. Then, in Section 3 the general class of dust interiors compatible with this exterior and with asymptotic safety is deduced. These solutions are analyzed in Section 4 in search of matter or curvature singularities. Finally, the results are discussed in the concluding Section 5.

2. Exterior: improved Schwarzschild solution

In order to model the gravitational collapse of *dust* in the Quantum Einstein Gravity approach to asymptotic safety we will assume

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the existence of a spherically symmetric spacetime $\mathcal{V}$ in which the collapse takes place. We will also assume that the spacetime will be split into two different regions $\mathcal{V} = \mathcal{V}^+ \cup \mathcal{V}^-$ with a common spherically symmetric time-like boundary $\Sigma = \partial\mathcal{V}^+ \cap \partial\mathcal{V}^-$, corresponding to the surface of the star. With regard to the stellar exterior region $\mathcal{V}^+$, we will describe it with a portion of an *improved Schwarzschild solution*. Specifically, we are choosing for the exterior region an effective improved solution coming from the asymptotic safety approach that incorporates quantum corrections to the classical solution ([9] and references therein). A summary of this effective solution could be the following. The spacetime metric for this solution can be written as

$$ds_+^2 = -\left(1 - \frac{2G(\bar{R})M}{\bar{R}}\right)dt_S^2 + \left(1 - \frac{2G(\bar{R})M}{\bar{R}}\right)^{-1} d\bar{R}^2 + \bar{R}^2 d\Omega^2. \tag{2.1}$$

where

$$G(\bar{R}) = \frac{G_0\bar{R}^3}{\bar{R}^3 + \tilde{\omega}G_0(\bar{R} + \gamma G_0 M)}, \tag{2.2}$$

G_0 is Newton's universal gravitational constant, M is the mass measured by an observer at infinity and $\tilde{\omega}$ and γ are constants coming from the non-perturbative renormalization group theory and from an appropriate cutoff identification, respectively. The qualitative properties of this solution are fairly insensitive to the precise value of γ. However, in [9,10] it is argued that the preferred value for γ is $\gamma = 9/2$. On the other hand, $\tilde{\omega}$ can be found by comparison with the standard perturbative quantization of Einstein's gravity (see [11] and references therein). It can be deduced that its precise value is $\tilde{\omega} = 167/30\pi$, but again the properties of the solution do not rely on its precise value as long as it is strictly positive.

If we define

$$\chi \equiv 1 - \frac{2G(\bar{R})M}{\bar{R}},$$

the horizons of the improved solution can be found by solving $\chi = 0$. It is easy to see that the horizons correspond to the number of positive real solutions of a cubic equation and depend on the sign of its discriminant or, equivalently, on whether the mass is bigger, equal or smaller than a critical value

$$M_{cr} = \frac{1}{24}\sqrt{\frac{1}{2}(2819 + 85\sqrt{1105})}\sqrt{\frac{\tilde{\omega}}{G_0}} \simeq 2.21\sqrt{\tilde{\omega}}m_p \simeq 2.94 m_p,$$

where m_p is Planck's mass. If $M > M_{cr}$ then the equation $\chi = 0$ has two positive real solutions $\{\bar{R}_I, \bar{R}_O\}$ satisfying $\bar{R}_I < \bar{R}_O$. The existence of an *inner* solution $\bar{R}_I$ represents a novelty with regard to the classical spacetime. However, it is interesting to remark that it is a result common to different approaches to Quantum Gravity. (See, for example, [12–14].) The *outer* solution $\bar{R}_O$ can be considered as the *improved Schwarzschild horizon*, i.e., the Schwarzschild horizon with quantum corrections taken into account. The 'improvement' in this horizon is, however, negligible for stellar masses, as can be made apparent if one expands $\bar{R}_O$ in terms of m_p/M obtaining

$$\bar{R}_O \simeq 2G_0 M\left[1 - \frac{(2+\gamma)}{8}\tilde{\omega}\left(\frac{m_p}{M}\right)^2\right].$$

In order to interpret the physical meaning of this solution let us suppose that it has been generated by an effective matter fluid in such a way that the coupled gravity-matter system satisfies Einstein's equations $G_{\mu\nu} = 8\pi G_0 T_{\mu\nu}$ [9,15]. Consider now a radially moving observer with an arbitrary 4-velocity $\bar{\mathbf{u}}$ and an orthonormal basis $\{\bar{\mathbf{u}}, \bar{\mathbf{n}}, \boldsymbol{\omega}_\theta, \boldsymbol{\omega}_\varphi\}$ such that $\boldsymbol{\omega}_\theta \equiv \bar{R}^{-1}\ \partial/\partial\theta$, $\boldsymbol{\omega}_\varphi \equiv (\bar{R}\sin\theta)^{-1}\ \partial/\partial\varphi$ and $\bar{\mathbf{n}}$ is a space-like 4-vector. The radially moving observer will write the vacuum energy–momentum tensor as an anisotropic fluid

$$\mathbf{T}^+ = \varrho_V \bar{\mathbf{u}} \otimes \bar{\mathbf{u}} + p_V \bar{\mathbf{n}} \otimes \bar{\mathbf{n}} + p_\perp(\boldsymbol{\omega}_\theta \otimes \boldsymbol{\omega}_\theta + \boldsymbol{\omega}_\varphi \otimes \boldsymbol{\omega}_\varphi), \tag{2.3}$$

where ϱ_V is the vacuum energy density, p_V is the vacuum normal pressure and $p_\perp$ is the vacuum tangential pressure. By using the field equations, one can obtain their explicit expressions:

$$\varrho_V = \frac{MG_{,\bar{R}}}{4\pi G_0 \bar{R}^2} = -p_V, \qquad p_\perp = -\frac{MG_{,\bar{R}\bar{R}}}{8\pi G_0 \bar{R}}, \tag{2.4}$$

where $G_{,\bar{R}}$ and $G_{,\bar{R}\bar{R}}$ are, respectively, the first and second derivatives of G with respect to $\bar{R}$.

3. Improved dust interiors

In order to obtain the complete stellar model, we are now searching for the general class of quantum improved interiors $\mathcal{V}^-$ made of non-interacting particles which are matchable with the improved exterior solution. In other words, $\mathcal{V}^+$and $\mathcal{V}^-$ should satisfy Darmois matching conditions on Σ, what implies that the interiors must be such that the first and second fundamental forms of Σ must coincide when computed from $\mathcal{V}^+$ or $\mathcal{V}^-$ [16,17].

Locally, every spherically symmetric spacetime metric can be written in geodesic coordinates as

$$ds_-^2 = -d\tau^2 + f(\tau,r)dr^2 + R(\tau,r)^2 d\Omega^2, \tag{3.1}$$

where, if the spacetime is filled with a fluid, τ is the proper time of the particles composing the fluid and r is a parameter that labels every shell of the fluid.

The matching of the interior solution to the improved Schwarzschild exterior will be performed through a spherically symmetric time-like hypersurface Σ comoving with the fluid. I.e., the stellar surface will be defined by choosing a matching shell $r = r_\Sigma$. Since we do not have energy entering or leaving the star, the total mass M of the star in the matched model should be completely determined by the value chosen for r_Σ, i.e., $M = M(r_\Sigma)$.

Darmois matching conditions and, in particular, the requirement that the first fundamental forms of Σ must coincide implies that the *areal radii* for the interior (R) and exterior regions ($\bar{R}$) must agree on Σ [17]:

$$R(\tau,r) \stackrel{\Sigma}{=} \bar{R}. \tag{3.2}$$

On the other hand, another consequence of the matching conditions is that the *mass functions* [18–20] at both sides of the matching hypersurface Σ must coincide [17]. The mass function of the interior solution is defined by $\mathcal{M}^- \equiv R(1 - g_-^{\alpha\beta}\partial_\alpha R \partial_\beta R)/(2G_0)$, what allows us to write f (for later use) as

$$f = \frac{R'^2}{\dot{R}^2 + 1 - 2G_0\mathcal{M}^-/R}, \tag{3.3}$$

where the apostrophe in R' denotes derivative with respect to r and the overdot in $\dot{R}$ denotes derivative with respect to τ. For the exterior, the mass function takes the form $\mathcal{M}^+ \equiv \bar{R}(1 - g_+^{\alpha\beta}\partial_\alpha \bar{R}\partial_\beta \bar{R})/(2G_0) = M(r_\Sigma)G(\bar{R})/G_0$, so that we will have on the matching surface

$$\mathcal{M}^-(\tau, r_\Sigma) \stackrel{\Sigma}{=} M(r_\Sigma)G(\bar{R})/G_0. \tag{3.4}$$