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Singularity free gravitational collapse in an effective dynamical quantum spacetime

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ABSTRACT

We model the gravitational collapse of heavy massive shells including its main quantum corrections. Among these corrections, quantum improvements coming from Quantum Einstein Gravity are taken into account, which provides us with an effective quantum spacetime. Likewise, we consider dynamical Hawking radiation by modeling its back-reaction once the horizons have been generated. Our results point towards a picture of gravitational collapse in which the collapsing shell reaches a minimum non-zero radius (whose value depends on the shell initial conditions) with its mass only slightly reduced. Then, there is always a rebound after which most (or all) of the mass evaporates in the form of Hawking radiation. Since the mass never concentrates in a single point, no singularity appears.

1. Introduction

It is expected that a large enough object would collapse classically until a horizon forms. Then Hawking radiation would appear and the mass of the object should be reduced. Less is known on the details of the later evolution. In fact, a complete investigation of the process would require a complete consistent theory of quantum gravity together with the calculational tools to achieve a description of the scenario. Since such apparatus is not currently available, for the moment one can only resort to the study of toy models in which the known main quantum contributions are taken into account. By this means, one can try to probe some of the features that one could expect from a full theory of quantum gravity.

In this Letter we work in this direction. In our toy model two main simplifications will be carried out. First, we assume the existence of a spherically symmetric spacetime $\mathcal{M}$ in which the collapse takes place. Second, we choose as our collapsing object a *thin shell*. In other words, we assume that the spacetime is split in two different regions $\mathcal{M} = \mathcal{M}^+ \cup \mathcal{M}^-$ with a common spherically symmetric timelike boundary $\Sigma = \partial\mathcal{M}^+ \cap \partial\mathcal{M}^-$ corresponding to the thin shell.

This second simplification deserves some comments. Clearly it means that we would be able to probe gravitational collapse only whenever the approximation in which one can neglect the shell thickness remains valid. An investigation of the conditions under which this is possible was carried out in [1] (see also [2]). The authors considered a shell composed of a number N of (s-wave) scalar particles with mass m bound together by gravitational interaction. The N particles form a radially localized bound state corresponding to a finite thickness shell, whose mean position approximately follows a classical collapse. Moreover, during the collapse the average shell thickness d decreases according to [1]

$$d \sim \frac{\hbar}{m}\left(\frac{R^2}{G_0(N-1)}\right)^{1/3},$$

where G_0 is Newton's gravitational constant. On the other hand, the fluctuations associated with the quantum nature of matter become dominant for a radius of the shell of the order of the Compton wavelength of the constituent quanta. So that they are negligible as long as [1]

$$R \gg \hbar/m.$$

In other words, if we want to probe the last stages of the collapsing phase by using the thin-shell approximation we should use a shell composed of a high number N of very heavy particles (large m's) and, thus, we would be using a heavy massive shell. Only under these conditions the results obtained using the thin-shell approximation are likely to be similar to the results that one would obtain in a real collapsing situation.[1]

With regard to the shell exterior region $\mathcal{M}^+$, we will describe it with a portion of an *improved Schwarzschild solution* with mass

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[1] On the contrary, a *light* shell with $M \lesssim m_p$ would possess a markedly quantum nature [3,4].

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equal to the shell mass. Specifically, we choose for the exterior region an effective improved solution coming from Quantum Einstein Gravity that incorporates quantum corrections to the classical solution. It does so by taking into account the effect of virtual gravitons. I.e., just as in quantum electrodynamics the virtual pairs imply the existence of a screening effect leading to a *running electric charge*, when one considers the existence of virtual gravitons one obtains an *antiscreening* effect leading to a *running gravitational constant*, which is used to get the improved Schwarzschild solution ([5] and references therein). A summary of this effective solution will be carried out in Section 2. On the other hand and following this approach, the shell massless interior region $\mathcal{M}^-$ will have to be described by a portion of Minkowski's spacetime (what is equivalent to a massless improved Schwarzschild solution).

Since the improved exterior solution possesses horizons, the tunneling of virtual particles through them is expected to produce Hawking radiation. Thus, in Section 3 we will also model the effect of the back-reaction to the radiation in the effective solution. Then, in Section 4 we will consider the matching of the interior and exterior solutions through the spherically symmetric thin shell Σ by using Israel's formalism [6]. This will provide us with the shell evolution equation which will allow us to analyze its different *attractive* and *repulsive* contributions. Finally in Section 5 the numerical integration of the evolution equation will be carried out and the results will be interpreted.

2. Exterior: improved Schwarzschild solution

As explained in the introduction, in order to model a collapsing shell we should first establish the exterior to that shell ($\mathcal{M}^+$). In this work we want the exterior to incorporate the main quantum corrections to the classical solution. This can be done by using a *renormalization group improved* Schwarzschild solution found by Bonanno and Reuter [5] that can be written as

$$ds^2 = -\left(1-\frac{2G(R)M}{R}\right)dt_S^2 + \left(1-\frac{2G(R)M}{R}\right)^{-1}dR^2 + R^2 d\Omega^2, \qquad (2.1)$$

where

$$G(R) = \frac{G_0 R^3}{R^3 + \tilde{\omega}G_0(R+\gamma G_0 M)}, \qquad (2.2)$$

G_0 is Newton's universal gravitational constant, M is the mass measured by an observer at infinity and $\tilde{\omega}$ and γ are constants coming from the non-perturbative renormalization group theory and from an appropriate "cutoff identification", respectively. The qualitative properties of this solution are fairly insensitive to the precise value of γ. In this way, in order to simplify the calculations, it is usual to choose $\gamma = 0$ [5,7]. On the other hand, $\tilde{\omega}$ can be found by comparison with the standard perturbative quantization of Einstein's gravity (see [8] and references therein). It can be deduced that its precise value is $\tilde{\omega} = 167/30\pi$, but again the properties of the solution do not rely on its precise value as long as it is strictly positive.

If we define

$$f \equiv 1 - \frac{2G(R)M}{R},$$

the horizons of the improved solution can be found by solving $f = 0$. Then, it is easy to see that the horizons correspond to the number of positive real solutions of a cubic equation and depend on the sign of its discriminant or, equivalently, on whether the mass is bigger, equal or smaller than a critical value M_{cr}. In particular, the value $\gamma = 0$ implies

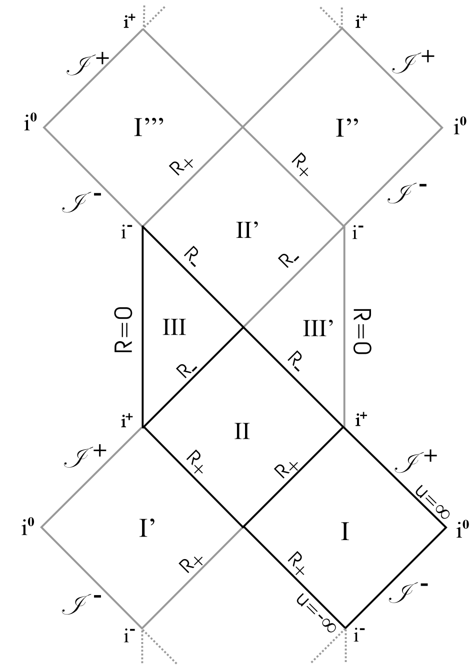

Fig. 1. A Penrose diagram corresponding to the case $M > M_{cr}$. The regions drawn using a solid black line (I–II–III) correspond to the zone defined by the solution in Eddington–Finkelstein-like coordinates (3.8) with the null coordinate going from $u = -\infty$ to $u = \infty$. The regions drawn in grey correspond to extensions of this solution.

$$M_{cr} = \sqrt{\frac{\tilde{\omega}}{G_0}} \simeq 1.33 m_p,$$

where m_p is the Planck mass. If $M > M_{cr}$ then the equation $f = 0$ has two positive real solutions $\{R_-, R_+\}$ satisfying $R_- < R_+$. The existence of an inner solution R_- represents a novelty with regard to the classical spacetime. However, it is interesting to remark that it is a result common to different approaches to Quantum Gravity. (See, for example, [5,9–11].) The outer solution R_+ can be considered as the *improved Schwarzschild horizon*, i.e., the Schwarzschild horizon with quantum corrections taken into account. The 'improvement' in this horizon can be made apparent for masses much bigger than Planck's mass if one expands R_+ in terms of m_p/M obtaining

$$R_+ \simeq 2G_0 M\left[1 - \frac{(2+\gamma)}{8}\tilde{\omega}\left(\frac{m_p}{M}\right)^2\right].$$

The global structure of the improved solution for $M > M_{cr}$ resembles the global structure of the Reissner–Nordström spacetime with mass bigger than its charge ($M > |Q|$). A Penrose diagram corresponding to the improved solution for the $M > M_{cr}$ case is shown in Fig. 1. On the other hand, if $M = M_{cr}$ then there is only one positive real solution to the cubic equation and the global structure resembles that of a extremal Reissner–Nordström solution ($M = |Q|$), whereas if $M < M_{cr}$ the equation has not positive real solutions.

3. Hawking radiation from the horizons

We will now summarize the results on Hawking radiation in the quantum improved solution. A more complete description can be found in [12,13] which, in turn, are based on the tunneling approach by Parikh and Wilczek [14]. We consider Hawking radiation coming out from an improved black hole satisfying $M > M_{cr}$