Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 658 (2008) 181–184
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Cosmological energy in a thermo-horizon and the first law

C. Barbachoux *, J. Gariel, G. Le Denmat

Observatoire de Paris, Université P. et M. Curie, LERMA, UMR CNRS 8112, ERGA, B.C. 142, 3, Rue Galilée, 94 200 Ivry, France

Received 3 May 2007; received in revised form 30 June 2007; accepted 27 July 2007

Available online 1 August 2007

Editor: M. Cvetič

Abstract

We consider a cosmological horizon, named thermo-horizon, to which are associated a temperature and an entropy of Bekenstein–Hawking and which obeys the first law for an energy flow calculated through the corresponding limit surface. We point out a contradiction between the first law and the definition of the total energy contained inside the horizon. This contradiction is removed when the first law is replaced by a Gibbs' equation for a vacuum-like component associated to the event horizon.

1. Introduction

The generalization of the thermodynamics of black holes (BH) to cosmological horizons represents an important take to understand different issues in cosmology such as the nature of the dark energy (DE) in relation with the problems of the cosmological constant (CC) and of the vacuum energy, the acceleration of the present universe, the coincidence problem and the early inflation.

This generalization was first introduced for de Sitter spacetime [1]. Thereafter, it was tentatively extended to quasi-de Sitter FRW spacetimes in different frameworks (see [2–6]). In an interesting approach, Bousso [7,8] considers the flow of energy through the horizon as a null surface. He interprets the variation of the entropy of the horizon through the variation of its surface as the response of the horizon to the flux of energy, in the same way as the "first law" of the BH.

Following this approach, several authors (see for example [9,10]) have estimated that the apparent horizon (a.h.) is the only limit surface (excluding other horizons such as the event horizon (e.h.)) having coherent thermodynamical properties to address problems such as the nature of the DE.

Our main goal is to shed some light on the contradiction between the amount of energy calculated from the first law as defined in [7,8] and the definition of the energy contained inside the horizon, independently of the choice of the thermo-horizon (t.h.).

We restrict our study to a spatially flat FRW spacetime, which is the starting point of other studies (non-spatially flat spacetimes, cases with interactions, ...).

After a brief review of the definition of a t.h. in a Q-space introduced in [7,8], we show that any t.h. obeys the second law (Section 2). In Section 3, we present the contradiction between the amount of energy derived from the first law and the definition of the energy inside the horizon. We then show that this contradiction is resolved in a thermodynamical model for a DE [4,5] based on the e.h. (Section 4).

2. Definition of a thermo-horizon

In a spatially flat FRW spacetime

$$ds^2 = -dt^2 + a(t)^2\big(dr^2 + r^2\, d\Omega^2\big), \tag{1}$$

the dynamical evolution of the scale factor $a(t)$ is given for a perfect fluid with energy density ρ and pressure P by

$$\left(\frac{\dot a}{a}\right)^2 = H^2 = \chi\frac{\rho}{3}, \tag{2}$$

$$\frac{\ddot a}{a} = -\frac{\chi}{6}(\rho + 3P), \tag{3}$$

where $\chi = 8\pi$ is the Einstein constant, with $G = 1$ and $c = 1$. The equation of state (EoS) ω of the fluid is given by $P = \omega\rho$

* Corresponding author.
E-mail address: barba@ccr.jussieu.fr (C. Barbachoux).

0370-2693 © 2007 Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2007.07.051

and we introduce the parameter $\varepsilon = \frac{3}{2}(1+\omega)$. In the following, we restrict our study to the Q-space [8], namely accelerated universes, for which $0 < \varepsilon < 1$. Extending the reasoning of [8], we consider an horizon (null surface) with a given radius L. According to the first law, the flow of energy through this surface is given by

$$-\dot{E} = 4\pi L^2 \rho(1+\omega) = T\dot{S}. \tag{4}$$

We assume that we can associate a temperature T and an entropy S to the dynamical horizon of radius L, given by the relations of Bekenstein–Hawking for a BH or a de Sitter horizon

$$T = \frac{1}{2\pi L} \quad \text{and} \quad S = \pi L^2. \tag{5}$$

Any horizon of radius L with a temperature and a entropy given by (5) and which obeys the first law (4) is called thermo-horizon (t.h.).

Using (5), we obtain directly $T\dot{S} = \dot{L}$ and Eq. (4) becomes

$$\varepsilon L^2 H^2 = \dot{L}. \tag{6}$$

With our notations, Eq. (3) is given by

$$\left(\dot{\frac{1}{H}}\right) = \varepsilon, \tag{7}$$

and the first law (6) rewrites

$$\dot{H} = \left(\dot{\frac{1}{L}}\right). \tag{8}$$

After integration, this equation leads to

$$HL - 1 = CL, \tag{9}$$

where C is a constant. Eq. (9) establishes a general relation between the t.h. L and the a.h. $R_A = \frac{1}{H}$ which is satisfied by any thermo-horizon of radius L without restriction on ε (in particular without assuming $\varepsilon = \text{const}$). With the constant C, this relation is more general than Eq. (28) of [8].

If L is the a.h., then $L = \frac{1}{H} = R_A$, implying $C = 0$. Conversely, only $C = 0$ leads to $L = \frac{1}{H}$. Therefore the a.h. obeys the first law (4) if and only if $C = 0$. This special case only is considered by [8].

More generally, any horizon L defined by (9) with a temperature and an entropy given by (5) is a t.h. and it obeys the first law (4).

Eq. (6) can be rewritten with the help of (9)

$$\dot{L} = \varepsilon(1+CL)^2. \tag{10}$$

Any t.h. verifies this equation. Using Eq. (10), L is strictly increasing in the Q-space (accelerated universe) where $0 < \varepsilon < 1$. The same result can be derived for the entropy S given by (5).

3. Energy in a thermo-horizon and the first law

On one side, the total amount of energy contained inside the a.h. for a spatially flat FW spacetime is (e.g. [9] before Eq. (20))

$$E = \rho \frac{4\pi}{3} R_A^3 = \frac{R_A}{2}. \tag{11}$$

Let us remark that in [10] this relation is used for a non-spatially flat FRW spacetime, albeit no more valid in this case.

On the other side, using (4) and (5), the first law applied to the a.h. considered as a t.h. leads to

$$-\dot{E} = \dot{R}_A, \tag{12}$$

where $-\dot{E}$ is the total amount of energy crossing the a.h. by unit of time. According to the conservation of the energy, this amount of energy is equal to the variation of the total energy (11) per unit time, $\dot{E} = \frac{\dot{R}_A}{2}$. This result is in contradiction with (12) except when R_A is constant, which corresponds to a de Sitter spacetime where the a.h. identifies with the e.h.

This result is not restricted to the a.h. and can be extended to any t.h. Using (7), the left hand side of (4) becomes for a t.h. of radius L,

$$-\dot{E} = L^2 H^2 \varepsilon = -L^2 \dot{H}, \tag{13}$$

while the total energy inside the horizon is

$$E = \frac{1}{2} H^2 L^3. \tag{14}$$

Differentiating (14) and equating with (13), we obtain with (8)

$$\frac{3}{2}(HL)^2 - HL + 1 = 0, \tag{15}$$

where $\dot{H} \neq 0$ has been assumed. No real root can be found for this equation. In particular, $L = \frac{1}{H}$ is not a solution. Therefore, the above contradiction can only be removed for $\dot{H} = 0$, namely for a de Sitter spacetime.

4. Thermodynamical model of the event horizon for the dark energy

The preceding results are independent of the underlying model for the DE. They depend only on the assumptions of the existence of a temperature and an entropy associated to a t.h. through the relation (5) and of the validity of the extension of the first law of BHs (4) to cosmological t.h. With these assumptions, we obtain (9) (assuming $C = 0$), as demonstrated by [7,8] and by [9], in Sections II-A and II-B.

In Section II-B of [9], the authors consider only the specific model for the DE developed in [6]. Let us emphasize that their results can be obtained independently of any model for the DE (see Section 2) because the demonstration involves only the density of the total energy ρ. Consequently, the reasoning developed in [9] cannot question the validity of the model assumed for the component DE and in particular the approach proposed in [6]. The first law (4) is a relation between the density of energy ρ and the entropy. It does not involve the density of energy of the DE ρ_Λ and therefore cannot be used to discuss or refute its expression.