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# Ten-dimensional super-Yang–Mills with nine off-shell supersymmetries

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**Abstract**

After adding 7 auxiliary scalars to the $d = 10$ super-Yang–Mills action, 9 of the 16 supersymmetries close off-shell. In this Letter, these 9 supersymmetry generators are related by dimensional reduction to scalar and vector topological symmetry in $\mathcal{N} = 2$, $d = 8$ twisted super-Yang–Mills. Furthermore, a gauge-invariant superspace action is constructed for $d = 10$ super-Yang–Mills where the superfields depend on 9 anticommuting $\theta$ variables.


## 1. Introduction

The off-shell field content of 10-dimensional super-Yang–Mills theory has an excess of seven fermionic degrees of freedom as compared to the number of gauge invariant bosonic degrees of freedom. To balance this mismatching, it was proposed in [1] to add to the supersymmetry transformation laws a set of seven auxiliary scalar fields $G_a$, together with a $\sum_a G_a^2$ term in the action. In order for the algebra to close off-shell, the parameters associated with the supersymmetry transformations must obey some identities. However, there is no linear solution to these identities, and thus no conventional supersymmetric formulation which permits the algebra to completely close.

It has been demonstrated in [1] that it is impossible to construct more than nine consistent solutions of these identities. Thus, only nine supersymmetry generators can generate an algebra that closes off-shell. These nine supersymmetry generators in ten-dimensional super-Yang–Mills are related to the octonionic division algebra in the same manner that the supersymmetry generators in three-, four-, and six-dimensional super-Yang–Mills are related to the real, complex, and quaternionic division algebras. However, the non-associativity of octonions makes the ten-dimensional supersymmetry algebra more complicated than in the other dimensions.

On the other hand, the $\mathcal{N} = 2$ twisted 8-dimensional super-Yang–Mills theory, which is a particular dimensional reduction of the 10-dimensional theory, has been determined in [2] by the invariance under a subalgebra of the maximal Yang–Mills supersymmetry. This subalgebra is small enough to close independently of equations of motion with a finite set of auxiliary fields, and yet is large enough to determine the Yang–Mills supersymmetric theory. It is also made of nine generators. The latter can be geometrically understood and constructed as scalar and vector topological Yang–Mills symmetries. This 8-dimensional topological symmetry can be built independently of the notion of supersymmetry, but, surprisingly, the latter symmetry with 16 generators can be fully recovered at the end of the construction.

The aim of this Letter is to make a bridge between the results of [1] and [2]. We will find that in 10-dimensional flat space with Lorentz group $SO(1,9)$ reduced to $SO(1,1) \times Spin(7)$, the supersymmetry algebra can be twisted such that the 10-dimensional super-Yang–Mills theory is determined by a supersymmetry algebra with 9 generators, which is related by

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dimensional reduction to the twisted $\mathcal{N}=2$ 8-dimensional super-Yang–Mills theory. Reciprocally, the extended curvature equation of the $\mathcal{N}=2$ 8-dimensional supersymmetric theory can be “oxidized” into an analogous 10-dimensional equation that determines the supersymmetry algebra and 10-dimensional super-Yang–Mills action. We argue that the largest symmetry group that can preserve an off-shell subalgebra of supersymmetry is $SO(1,1)\times Spin(7)$, and we obtain the most general $SO(1,1)\times Spin(7)$ covariant solution of the identities defined in [1]. The supersymmetry algebra that we derive is exactly the one obtained by the twist operation.

We then define a superspace involving nine Grassmann $\theta$ variables such that the off-shell supersymmetry subalgebra acts in a manifest way on the super-Yang–Mills superfields. Using these off-shell superfields, a superspace action is constructed which reproduces the ten-dimensional super-Yang–Mills action including the seven auxiliary scalar fields $G_a$. Although this superspace action is manifestly invariant under only a $Spin(7)\times SO(1,1)$ subgroup of $SO(9,1)$, it is manifestly invariant under nine supersymmetries as well as gauge transformations. This can be compared with the light-cone superspace action for ten-dimensional super-Yang–Mills which is manifestly invariant under eight supersymmetries and an $SO(8)\times SO(1,1)$ (or $U(4)\times SO(1,1)$) subgroup of $SO(9,1)$, but is not manifestly invariant under gauge transformations.

## 2. Ten-dimensional supersymmetric Yang–Mills with auxiliary fields

The Poincaré supersymmetric Yang–Mills theory in ten-dimensional Minkowski space contains a gauge field $A_\mu$ ($\mu=1,\dots,10$) and a sixteen-component Majorana–Weyl spinor $\Psi$, with values in the Lie algebra of some gauge group. In order to balance the gauge-invariant off-shell degrees of freedom, one can introduce a set of scalar fields $G_a$ ($a=1,\dots,7$) which count for the 7 missing bosonic degrees of freedom [1]. The Lagrangian is given by

$$\mathcal{L}=\mathrm{Tr}\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{i}{2}\left(\bar\Psi\hat\Gamma^\mu D_\mu\Psi\right)+8G_aG_a\right\},\tag{1}$$

where $\hat\Gamma^\mu$ are the ten-dimensional gamma matrices. As shown in [1], the action (1) is invariant under the following supersymmetry transformations, which depend on the ordinary Majorana–Weyl parameter $\epsilon$ and on seven other spinor parameters $v_a$

$$\delta A_\mu=i\bar\epsilon\hat\Gamma_\mu\Psi,$$
$$\delta\Psi=\hat\Gamma^{\mu\nu}F_{\mu\nu}\epsilon+4G_av_a,\tag{2}$$
$$\delta G_a=-\frac{i}{4}\bar v_a\hat\Gamma^\mu D_\mu\Psi.\tag{3}$$

The commuting spinor parameters $v_a$ must be constrained as follows

$$\bar v_a\hat\Gamma_\mu\epsilon=\bar v_a\hat\Gamma_\mu v_b-\delta_{ab}\bar\epsilon\hat\Gamma_\mu\epsilon=0.\tag{4}$$

The transformations (2) generate a closed algebra modulo gauge transformations and equations of motion

$$\{\delta,\hat\delta\}\approx-2i\bar\epsilon\hat\Gamma^\mu\hat\epsilon\,\partial_\mu-2i\delta^{\text{gauge}}\left(\bar\epsilon\hat\Gamma^\mu A_\mu\hat\epsilon\right)\tag{5}$$

and close independently of equations of motion when

$$(\hat\epsilon,\hat v_a)\tag{6}$$

is some linear combination of $(\hat\Gamma^{\mu\nu}\epsilon,\hat\Gamma^{\mu\nu}v_a)$. To recover conventional supersymmetry transformations, one must have a solution for $v$ in (4) that is linear in $\epsilon$. This in turn will give a realization of (5) which, thanks to (6), will effectively hold off-shell.

Using octonionic notations and light-cone coordinates, a solution was found for the $v$'s and $\epsilon$ in [1] that preserves nine supersymmetries. This solution is only covariant under $SO(1,1)\times Spin(7)\subset SO(1,9)$. In fact, in order to define the $v$'s as linear combinations of $\epsilon$, we must reduce the covariance to a subgroup $H$ that admits a 7-dimensional representation. Moreover, since the maximal sub-algebra that can be closed off-shell contains 9 supersymmetry generators, the Majorana–Weyl spinor representation of $Spin(1,9)$ must decompose into $\mathbf{7}+\mathbf{9}$ of $H$. The biggest subgroup of $SO(1,9)$ that satisfies these criteria is $SO(1,1)\times Spin(7)$.

### 2.1. Light-cone variables

The choice of light-cone variables implies a reduction of the Lorentz group as

$$SO(1,9)\to SO(8)\times SO(1,1),\tag{7}$$

where the spinor $\Psi\in\mathbf{16}_+$ of $SO(1,9)$ decomposes into one chiral and one antichiral spinor of $Spin(8)$, $\Psi\to\lambda_1\oplus\lambda_2\in\mathbf{8}_+^{-1}\oplus\mathbf{8}_-^{1}$, as well as $\epsilon\to\epsilon_1\oplus\epsilon_2$ and $v_a\to v_{a1}\oplus v_{a2}$. The connection $A_\mu\in\mathbf{10}$ of $SO(1,9)$ decomposes according to $A_\mu\to A_i\oplus A_+\oplus A_-\in\mathbf{8}_\mathbf{v}^\mathbf{0}\oplus\mathbf{1}^\mathbf{2}\oplus\mathbf{1}^{-\mathbf{2}}$ of $SO(8)\times SO(1,1)$, where the superscripts denote the eigenvalue associated with the $SO(1,1)$ factor and $A_\pm=A_0\pm A_9$.

We can consider a gamma matrix algebra of $Cl(1,9)$ in terms of gamma matrices of $Cl(0,8)$

$$\hat\Gamma_0=(i\sigma_2)\otimes\Gamma_9,$$
$$\hat\Gamma_i=\sigma_2\otimes\Gamma_i\quad(i=1,\dots,8),$$
$$\hat\Gamma_9=\sigma_1\otimes 1\tag{8}$$

and $\hat\Gamma_{11}\Psi=\sigma_3\otimes 1\,\Psi=\Psi$. These matrices obey $[\hat\Gamma_\mu,\hat\Gamma_\nu]=2\eta_{\mu\nu}$, with the metric $\eta_{\mu\nu}=\mathrm{diag}(-1,+1,\dots,+1)$ and $[\Gamma_i,\Gamma_j]=2\delta_{ij}$. The decomposition $SO(1,9)\to SO(8)\times SO(1,1)$ is performed by taking $A_\pm=A_0\pm A_9$ and by projecting $\Psi$ to $\hat\Gamma_+\Psi\to\lambda_2$ and $\hat\Gamma_-\Psi\to\lambda_1$, with $\Gamma_9\lambda_1=\lambda_1$ and $\Gamma_9\lambda_2=-\lambda_2$.

The transformations laws (2) read

$$\delta A_i=-i\bar\epsilon\Gamma_i\lambda,$$
$$\delta A_+=-2\bar\epsilon_2\lambda_2,$$
$$\delta A_-=2\bar\epsilon_1\lambda_1,$$
$$\delta\lambda_1=\left(F_{ij}\Gamma_{ij}+\frac{1}{2}F_{+-}\right)\epsilon_1+iF_{i-}\Gamma_i\epsilon_2+4G_av_{a1},$$