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Charged condensation

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Abstract

We consider Bose–Einstein condensation of massive electrically charged scalars in a uniform background of charged fermions. We focus on the case when the scalar condensate screens the background charge, while the net charge of the system resides on its boundary surface. A distinctive signature of this substance is that the photon acquires a Lorentz-violating mass in the bulk of the condensate. Due to this mass, the transverse and longitudinal gauge modes propagate with different group velocities. We give qualitative arguments that at high enough densities and low temperatures a charged system of electrons and helium-4 nuclei, if held together by laboratory devices or by force of gravity, can form such a substance. We briefly discuss possible manifestations of the charged condensate in compact astrophysical objects.

1. Introduction and summary

Consider a sphere enclosing massive stable charged spin-1/2 particles with number density $\bar{J}_0$, and stable massive spin-0 particles of an equal but opposite charge. At some high temperature the substance in the sphere could form hot plasma. With the decreasing temperature the opposite charges would ordinarily form neutral atoms of half-integer spins. These atoms would not be able to Bose–Einstein condense because of their spin-statistics.

We will discuss in this work a different sequence of events that could take place in the above system. In particular, we will show that under certain conditions, instead of forming neutral atoms, the charged scalars could themselves condense, neutralizing by this condensate the background charge of the fermions.

Especially interesting we find the case when the system has a net overall charge to begin with. In this case, although the resulting substance is charge neutral in the interior of the sphere, the net charge will reside on its surface. The substance in the bulk has distinctive properties. We will show in Section 2 that propagation of a photon in this substance is rather special. Even at zero temperature, the photon acquires a Lorentz non-invariant mass term. The transverse and longitudinal components of the photon have equal masses; the mass squares are proportional to $\bar{J}_0$ and inversely proportional to the charged scalar mass. However, the group velocities of the transverse and longitudinal modes are different. The longitudinal mode is similar to a plasmon excitation of cold plasma. The transverse modes of the photon propagate as massive states. We will refer to this phase as the charged condensate, emphasizing that the charged scalars have undergone Bose–Einstein condensation, while the background fermions merely play the role of charge neutralizers in the bulk of the substance, and the net charge of the system is residing on the boundary.

The above mechanism is universal: the gauge field could be a photon or any other $U(1)$ field, while the charged scalar could be a fundamental field, or a composite state made of other particles, in the regime where its compositness does not matter. This may have applications in particle physics and condensed matter systems.

As a concrete example we imagine a reservoir, or a trap, in which negatively charged electrons and positively charged helium-4 nuclei, with a nonzero net charge, could be put together at densities high enough for an average inter-particle separation to be smaller than the size of a helium atom. In this case, the helium atoms would not form. The results of Section 2 cannot immediately be applied to this case, since electrons are lighter than the helium nuclei. However, we will argue in Sec-

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tion 3 that if temperature of the system is low enough for the helium de Broglie wavelength to be greater than both the average inter-particle separation and the Compton wavelength of the massive photon, then the charged helium-4 nuclei would fall into the condensate. Photons, in the bulk of this substance, would propagate with a delay caused by the acquired mass. Such a system would also have a net surface charge. Quantitative features of this example are discussed in Section 3. Our estimate for the temperature is within the range of the low temperatures that have already been achieved in experiments on Bose–Einstein condensation of atoms, see, e.g., [1].

In the above example the charged condensate containing droplet was assumed to be held together by a rigid boundary or external fields in a laboratory. In Section 4 we point out that gravity could play the role of the stabilizing force, and briefly discuss possible manifestations of the charged condensation in compact astrophysical objects.

A few comments on the literature. The pion condensation due to strong interactions is well known [2]. In this work we discuss condensations due to electromagnetic interactions instead (or in more general case, due to some $U(1)$ Abelian interactions). It was shown in Ref. [3] that the constant charge density strengthens spontaneous symmetry breaking when the symmetry is already broken by the usual Higgs-like nonlinear potential for the scalar. In our work the scalar has a conventional positive-sign mass term. The fact that the conventional-mass scalar could condense in the charged background was first shown in [4]. However, the system considered in [4] is neutral, and thus, is physically different from the one studied in this work (see, brief comments after Eq. (4.6) in [4]). An expanded discussions of the topics covered in the present work, with other possible applications will be presented elsewhere [5].

2. Basic mechanism

We consider a simplest model that exhibits the main phenomenon. Let us start with a system in an infinite volume and at zero-temperature. The classical Lagrangian contains a gauge field A_μ, a charged scalar field ϕ with a right-sign mass term $m_H^2 > 0$, and fermions Ψ^+, Ψ with mass m_J

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu\phi|^2 - m_H^2\phi^*\phi + \bar{\Psi} i\gamma^\mu D_\mu \Psi - m_J\bar{\Psi}\Psi + \mu\Psi^+\Psi. \tag{1}$$

The chemical potential μ is introduced for the global fermion number carried by Ψ's (e.g., lepton, baryon or other number). The covariant derivatives in (1) are defined as $\partial_\mu + ig_\phi A_\mu$ for the scalars, and $\partial_\mu + ig_\psi A_\mu$ for the fermions. Their respective charges, g_ϕ and g_ψ, are different in general. For simplicity we assume that $g_\phi = -g_\psi \equiv -g$.

To study the ground state it is convenient to introduce the following notations for the scalar, gauge field and fermions: $\phi = \frac{1}{\sqrt{2}}\sigma e^{i\alpha}$, $B_\mu \equiv A_\mu + \frac{1}{g}\partial_\mu\alpha$, and $\psi = \Psi e^{-i\alpha}$. In terms of the gauge invariant variables σ, B_μ and ψ the Lagrangian, takes the form

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}g^2B_\mu^2\sigma^2 - \frac{1}{2}m_H^2\sigma^2 + \bar{\psi} i\gamma^\mu D_\mu\psi - m_J\bar{\psi}\psi + \mu\psi^+\psi, \tag{2}$$

where now $F_{\mu\nu}$ and D are a field-strength and covariant derivative for B_μ, respectively.

Fermions in (2) obey the conventional Dirac equation with a nonzero chemical potential. This implies a net fermion number in the system, $\bar{J}_0$. Since the fermions are also electrically charged, they set a background electric charge density. Such charged fermions would repel each other. In our case, however, the charge will be screened by the charged scalar condensate. One way to see this is to assume that such a self-consistent solution exists, and then check explicitly that it satisfied equations of motion, as we will do it below. We consider distance scales that are greater than an average separation between the fermions, so that their spatial distribution could be assumed to be uniform. Then, the background charge density due to the fermions could be approximated as $\bar{J}_\mu = \bar{J}_0\delta_{\mu 0}$, where $\bar{J}_0$ is a constant. The magnitude of the latter is related to the value of the chemical potential μ. In particular, a self-consistent solution of the equations of motion implies that $\mu - \langle gB_0\rangle = E_F$, where E_F denotes the Fermi energy of the background fermion sea, and is related to $\bar{J}_0$ as follows, $E_F = \sqrt{(3\pi\bar{J}_0/4)^{2/3} + m_J^2}$.

The rest of the equations of motion derived from (2) are:

$$\partial^\mu F_{\mu\nu} + g^2B_\nu\sigma^2 = g\bar{J}_\nu, \qquad \Box\sigma = g^2B_\nu^2\sigma - m_H^2\sigma. \tag{3}$$

The Bianchi identity for the first equation in (3), $\partial^\nu(B_\nu\sigma^2) = 0$, can also be obtained by varying the action w.r.t. α. For a constant charge density, $\bar{J}_\mu = \bar{J}_0\delta_{\mu 0}$, the theory with the scalar field (1) admits a static solution with constant B_0 and σ:

$$\langle B_0\rangle = B_{0c} \equiv \frac{m_H}{g}, \qquad \langle\sigma\rangle = \sigma_c \equiv \sqrt{\frac{\bar{J}_0}{m_H}}. \tag{4}$$

The charge density stored in the condensate, $J_0^{\text{scalar}} = -i[\phi^* D_0\phi - (D_0\phi)^*\phi] = -g\sigma^2 B_0$, equals to $-\bar{J}_0$, by virtue of (4). Hence, the total charge density $J_{\text{total}} = \bar{J}_0 + J_0^{\text{scalar}} = 0$, vanishes. The ground state is charge-neutral in its bulk. On the other hand, a nonzero $\langle B_0\rangle$ in (4) suggests that there must be an uncompensated charge on a surface at infinity, as it will be the case (see below).

Before we continue with studies of small perturbations about the solution (4), we would like to make four essential comments:

(i) The expression for the gauge field in (4) scales as $1/g$, and is non-perturbative in its nature. Moreover, it diverges in the limit $m_H \to \infty$. This seeming non-decoupling of the charged scalar field results from the fact that we are dealing with a constant background *charge density* in an infinite volume, i.e., with an infinite background charge. It is not surprising then, that such a background is capable of affecting a charged state of an arbitrary mass. Moreover, when m_H exceeds the fermion mass, our averaging procedure over the background charges should not be applicable in general.

(ii) In regard with the above discussions, it is instructive to regularize the problem by considering a finite volume ball of a