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On regularizing the infrared singularities in QCD NLO splitting functions with the new Principal Value prescription

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ABSTRACT

We propose a modified use of the Principal Value prescription for regularizing the infrared singularities in the light-cone axial gauge by applying it to all singularities in the light-cone plus component of integration momentum. The modification is motivated by and applied to the re-calculation of the QCD NLO splitting functions for the purpose of Monte Carlo implementations. The final results agree with the standard PV prescription whereas contributions from separate graphs get simplified.

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1. Introduction

With the advent of the precision QCD measurements from the LHC there is an interest in the re-calculation of the QCD splitting functions at the NLO level, either in order to construct more precise, exclusive, Parton Shower Monte Carlo (MC) algorithms or to improve the convergence of the logarithmic expansion of PDFs [1–4]. The physical interpretation of the evolution, necessary to construct the Parton Shower MC, is best visible in the axial gauge in which the NLO calculations have been done [5–8]. A price to pay for the transparent physical picture is the appearance of the spurious singularities associated with the axial denominator $1/(nl)$ where n is the light-cone reference vector. These unphysical singularities cancel at the end, but in the intermediate stages of the calculations one has to somehow regularize them. The simplest way is to use the Principal Value (PV) prescription [5,6,9,10]. The other option is the Mandelstam–Leibbrandt (ML) prescription [11,12], which is better justified from the field-theoretical point of view, but leads to more complicated calculations, especially for the real-emission-graphs [9]. Other methods of avoiding the problem of spurious singularities can be found in [13,14].

The standard PV regularization is applied at the level of the Feynman rules to the axial part of the gluon propagator:

$$g^{\mu\nu} - \frac{l^\mu n^\nu + n^\mu l^\nu}{nl} \to g^{\mu\nu} - \frac{l^\mu n^\nu + n^\mu l^\nu}{[nl]_{PV}},$$
$$\left[\frac{1}{nl}\right]_{PV} = \frac{nl}{(nl)^2 + \delta^2 (pl)^2} \tag{1}$$

where p is an external reference momentum and δ is an infinitesimal regulator of the "spurious" singularities. Spurious singularities are artifacts of the gauge choice and are expected to cancel completely once the full set of graphs is added. On the other hand, apart from the axial part of the propagators, there are also other singularities in the $l_+ = nl$ variable, associated with the Feynman part of the propagator ($g^{\mu\nu}$) or phase space parametrization. In the standard approach [5–8] these singularities are regularized by means of dimensional regularization. As a consequence, in the final results for single graphs we encounter both $\ln^2\delta$ and $1/\epsilon^2$ terms. This complicates calculations as well as makes results useless for the stochastic simulations, which are supposed to be done in four dimensions.

In this note we propose a new way of using the PV regularization. We show that the proposed scheme, called the NPV scheme, reproduces the QCD NLO splitting functions correctly and in a simpler way. The contributions from separate graphs are less singular

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in ϵ, at the expense of increased PV-regulated singularities,[1] and the remaining higher order singularities in most cases cancel separately in real and virtual groups of diagrams.

2. New PV prescription

We propose to modify the PV prescription as follows: apply the PV regularization of Eq. (1) to *all* the singularities in the plus variable, not only to the axial denominators of the gluon propagators, i.e. we propose to replace

$$d^m l\, l_+^{-1+\epsilon} \to d^m l \left[\frac{1}{l_+}\right]_{PV} \left(1 + \epsilon \ln l_+ + \epsilon^2 \frac{1}{2} \ln^2 l_+ + \cdots\right), \qquad l_+ = \frac{nl}{np} \tag{2}$$

in the entire integrand and we keep the PV regulator δ small but finite until the end of calculation. The higher order ϵ terms are kept as needed. In the following we will denote this new scheme as the NPV prescription.

The standard procedure of introducing Feynman parameters, integrating out m-momentum and at last integrating out Feynman parameters, is not suitable for calculations in NPV scheme. Instead, one has to isolate the integral over the plus component of m-momentum and leave it for the very end. The appropriate formulae are available in the literature [6] (see [9] for details of derivation). Let us quote here Eq. (A.12) of [6] for the three-point integral with the kinematics $p^2 = (p-q)^2 = 0$, expanded to $\mathcal{O}(\epsilon^0)$ terms

$$\begin{aligned}
&\int \frac{d^m l}{(2\pi)^m} \frac{f(l_+)}{l^2 (l-q)^2 (l-p)^2} \\
&= \frac{-i}{16\pi^2 q^2} \left(\frac{4\pi}{-q^2}\right)^{-\epsilon} \frac{\Gamma(1-\epsilon)}{-\epsilon} \\
&\times \left[\int_0^x dy\, f(l_+) z^\epsilon (1-z)^\epsilon \left(1 + 2\epsilon \ln \frac{1-y}{1-z}\right) \frac{1}{1-y} \right. \\
&\left. + 2 \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} (1-x)^{-\epsilon} \int_x^1 dy\, f(l_+) (1-y)^{-1+2\epsilon} \right], \\
&m = 4 + 2\epsilon,
\end{aligned} \tag{3}$$

where $x = q_+/p_+$, $y = l_+/p_+$, $z = y/x = l_+/q_+$ and $f(l_+)$ is an arbitrary function of the plus variable. The PV prescription enters through this function, which can have end-point singularities at $y = 0$ or $y = x$. There is however also a singularity at $y = 1$, in the last but one line of Eq. (3), not related to the axial function f. It is this singularity that is treated differently: in the standard PV prescription it simply reads $(1-y)^{-1+2\epsilon}$, whereas in our NPV one it is also regularized with PV and becomes $(1-y)^{2\epsilon}[1/(1-y)]_{PV}$. As a consequence even non-axial three point integrals are changed and start to depend on the auxiliary vector n. Consider, for example, the scalar integral

$$J_3^{\mathrm{F}} = \int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{1}{l^2 (q-l)^2 (p-l)^2} \tag{4}$$

with the kinematical set-up: $p^2 = (p-q)^2 = 0$, $q^2 < 0$. The PV regularization gives:

$$J_3^{\mathrm{F}} = \frac{i}{(4\pi)^2 |q^2|} \left(\frac{4\pi}{|q^2|}\right)^{-\epsilon} \Gamma(1-\epsilon) \left(-\frac{1}{\epsilon^2} + \frac{\pi^2}{6}\right), \tag{5}$$

whereas the new NPV prescription leads to:

$$\begin{aligned}
J_3^{\mathrm{F}} &= \frac{i}{(4\pi)^2 |q^2|} \left(\frac{4\pi}{|q^2|}\right)^{-\epsilon} \Gamma(1-\epsilon) \left(-\frac{2I_0 + \ln(1-x)}{\epsilon} \right. \\
&\quad \left. - 4I_1 + 2I_0 \ln(1-x) + \frac{\ln^2(1-x)}{2}\right), \\
I_0 &= \int_0^1 dx \frac{1}{[x]_{PV}} = -\ln\delta + \mathcal{O}(\delta), \\
I_1 &= \int_0^1 dx \frac{\ln x}{[x]_{PV}} = -\frac{1}{2}\ln^2\delta - \frac{\pi^2}{24} + \mathcal{O}(\delta),
\end{aligned} \tag{6}$$

where $x = q_+/p_+$ is the axial-vector-dependent parameter. As expected, Eq. (6) is free of double poles in ϵ. Instead, the I_0/ϵ and I_1 functions appeared. The list of other three-point integrals needed for calculations of the NLO splitting functions in the NPV scheme is given in Appendix A.

Discussion: The use of the PV prescription has been criticized for a lack of a solid field-theoretical basis, for example, for not preserving the causality [15,8]. On the other hand, it leads to correct results for the NLO splitting functions [5,8]. The singularities in n-direction are unphysical (spurious). As such, they must cancel at the end of the calculation once all the graphs are included. Therefore, as argued in [5], one can employ a "phenomenological" PV recipe of how to deal with them in the intermediate steps of the calculations. The proposed here new NPV scheme follows the same justification: the "non-spurious" IR singularities in plus-variable also cancel once the whole set of graphs entering NLO splitting functions is added [16]. Therefore it is natural to extend the PV regularization and treat all the singularities of the plus-variable on an equal footing. Let us remark that separate regularization of the energy component of the loop momentum is a known approach. For example in [17] the singularities of the Coulomb gauge have been regularized by means of "split dimensional regularization" in which the measure $d^m l$ is replaced by $d^{2(\sigma+\omega)} l = d^{2\sigma} l_0 d^{2\omega} \vec{l}$.

On the technical level the NPV prescription simplifies the calculations – one does not need to keep two types of regulators for the higher order poles. In the real emission case the triple and double poles in ϵ vanish, replaced by $\ln\delta$, the calculations can be done in four dimensions [10] and there is no need of cancelling these higher order poles between real and virtual graphs. The price to pay for these simplifications is that the non-axial integrals become more complicated, as can be seen by comparing Eqs. (5) and (6).

3. NLO splitting functions in the NPV scheme

We are going to demonstrate now how the NPV scheme works in the calculations of the NLO quark–quark and gluon–gluon splitting functions and we show that it reproduces the known final results of PV prescription [5,18]. More detailed results in the standard PV prescription can be found in [9,6]. One can see there that in the standard PV prescription the triple poles, $1/\epsilon^3$, appear only in the four real and virtual interference graphs of the type "(*d*)", shown in Fig. 1, both for the non-singlet and singlet cases and only these graphs will be affected by the change from PV to NPV prescription because the other, lower, ϵ poles come from transverse- or minus-components of integration momenta. We will discuss in turn these four contributions using the standard formula relating the graphs Γ and splitting functions P:

[1] PV regularization is directly implementable in the MC computer codes.

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