Physics Letters B 732 (2014) 127–136

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Novel collective excitations in a hot scalar field theory

Marco Drewes

Physik Department T70, Technische Universität München, James Franck Straße 1, D-85748 Garching, Germany

ARTICLE INFO

Article history:
Received 20 December 2013
Received in revised form 18 February 2014
Accepted 11 March 2014
Available online 17 March 2014
Editor: A. Ringwald

ABSTRACT

We study the spectrum of quasiparticles in a scalar quantum field theory at high temperature. Our results indicate the existence of novel quasiparticles with purely collective origin at low momenta for some choices of the masses and coupling. Scalar fields play a prominent role in many models of cosmology, and their collective excitations could be relevant for transport phenomena in the early universe.

1. Introduction

Quantum field theory provides the most fundamental description of matter and radiation we know and solves the apparent "wave particle dualism" in a consistent way. With quantised fields being the fundamental building blocks of nature, the elementary excitations of these fields in weakly coupled systems propagate like particles. Curiously, there are not only elementary particles; in a medium the collective excitations of many elementary quanta often effectively behave as if they were particles themselves.

Often one is not interested in the fate of individual particles, but mostly in transport of energy or charges within a system. Transport phenomena can be studied in a thermodynamic description in terms of a density matrix ϱ. The propagator in this effective thermodynamic description can have a rather different structure than in vacuum. This reflects the fact that propagating particles are affected by the medium. In weakly coupled systems this effect can often be parametrised by interpreting the poles of the propagator as *quasiparticles* with modified properties. For instance, the dispersion relations (or "bands") of electrons in a solid state can be very different from that in vacuum. Also the effective charge is screened in a medium. In addition to the screened elementary particles there can be new types of quasiparticles that have no analogue in vacuum. These can be interpreted as quantised collective excitations of the background medium. For instance, in a solid state the lattice vibrations, phonons, behave like quasiparticles. The existence of collective excitations is also well-known from relativistic quantum field theory. In gauge theories with coupling $\alpha \ll 1$ in thermal equilibrium at high temperature T there are fermionic excitations with soft momenta $\mathbf{p} \sim \alpha T$ [1–5] and ultrasoft momenta $\mathbf{p} \sim \alpha^2 T$ [6,7] which have no analogue in vacuum. These are often referred to as *holes* or *plasminos*. Collective fermionic excitations have also been found in models with Yukawa interactions [8–10]. Also longitudinal gauge bosons appear at finite temperature with a dispersion relation that differs from the transverse components.

In this work we find evidence that collective excitations can also exist in purely scalar field theories. The existence of collective propagating modes in principle is expected; in particular hydrodynamic modes, such as sound waves, should appear in the spectrum of any field theory. However, to the best of our knowledge, quasiparticles beyond the hydrodynamic regime have not been described explicitly in the context of purely scalar field theories. On one hand, their existence can simply be viewed as an interesting property of the field theory. On the other hand, current experimental evidence [11,12] suggests that there is at least one scalar field in nature, the Higgs field. Furthermore, many models of cosmology involve additional scalar fields, such as axions, the inflaton, dilaton, moduli fields or Affleck–Dine fields. Since the universe was exposed to very high temperatures during the early stages of its history, the spectrum of scalar quasiparticles may have affected transport phenomena in the early universe.

2. The quasiparticle spectrum in a simple scalar model

We consider a simple model of two scalar fields described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2 - g\phi\chi^2. \quad (1)$$

We choose this Lagrangian for illustrative purposes, as it describes the (probably) simplest scalar model in which the additional collective excitations we found appear. We expect that similar behaviour can be found in more realistic models where the structure of the self-energies is similar.[1]

[1] Note that the energy functional obtained from (1) is not bound from below. For the purpose of illustrating the appearance of collective scalar quasiparticles we will

http://dx.doi.org/10.1016/j.physletb.2014.03.019

2.1. Quasiparticles in thermal field theory

Following the approach of [13–15] we study the system in terms of real time correlation functions. This approach has been applied to scalar fields in different situations [16–31] relevant for cosmology. We use the notation of [20]. The expectation values or one-point functions $\langle\phi(x)\rangle$ and $\langle\chi(x)\rangle$ play the role of the "classical field". The average $\langle\cdots\rangle$ is defined in the usual way as $\langle\mathcal{A}\rangle = \mathrm{Tr}(\varrho\mathcal{A})$, where ϱ is the density matrix of the thermodynamic ensemble. It includes the usual quantum average as well as a statistical average over initial conditions. We will in the following assume that all degrees of freedom are in thermal equilibrium and set $\langle\phi(x)\rangle = \langle\chi(x)\rangle = 0$. Quasiparticle properties are encoded in the propagator or two-point function. We can define two independent two-point functions for ϕ,

$$\Delta^-(x_1,x_2) = i\big(\langle\phi(x_1)\phi(x_2)\rangle - \langle\phi(x_2)\phi(x_1)\rangle\big) \quad (2)$$

$$\Delta^+(x_1,x_2) = \frac{1}{2}\big(\langle\phi(x_1)\phi(x_2)\rangle + \langle\phi(x_2)\phi(x_1)\rangle\big), \quad (3)$$

and analogously for χ. Δ^- is called the *spectral function*. It encodes the properties of quasiparticles and is the main quantity of interest in this work. Δ^+ is called the *statistical propagator* and characterises the occupation numbers of different modes. Out of thermal equilibrium, $\Delta^-(x_1,x_2)$ and $\Delta^+(x_1,x_2)$ would be two independent functions, and each of them would depend on x_1 and x_2 individually. Thermal equilibrium is homogeneous, isotropic and time translation invariant, hence the correlation functions can only depend on the relative coordinate $x_1 - x_2$.[2] This allows to define the Fourier transform

$$\rho_{\mathbf{p}}(p_0) = -i\int d^4(x_1-x_2)\; e^{ip_0(t_1-t_2)}e^{-i\mathbf{p}(\mathbf{x}_1-\mathbf{x}_2)}\Delta^-(x_1-x_2). \quad (4)$$

It can be expressed as [20]

$$\rho_{\mathbf{p}}(p_0) = \frac{-2\,\mathrm{Im}\,\Pi^R_{\mathbf{p}}(p_0) + 2p_0\epsilon}{(p_0^2 - m^2 - \mathbf{p}^2 - \mathrm{Re}\,\Pi^R_{\mathbf{p}}(p_0))^2 + (\mathrm{Im}\,\Pi^R_{\mathbf{p}}(p_0) + p_0\epsilon)^2}. \quad (5)$$

Here $\Pi^R_{\mathbf{p}}(p_0)$ is the Fourier transform of the usual retarded self-energy, in this case

$$\Pi^R_\phi(x_1,x_2) = g^2\theta(t_1-t_2)\big(\chi(x_1)\chi(x_1)\chi(x_2)\chi(x_2) - \chi(x_2)\chi(x_2)\chi(x_1)\chi(x_1)\big), \quad (6)$$

and analogous for χ. In (5) we have not specified whether we refer to ϕ or χ; both spectral densities formally have the same shape except for the replacement $m \to m_\phi$ or $m \to m_\chi$ and the insertion of the corresponding self-energy. The pole structure of $\rho_{\mathbf{p}}(p_0)$ in the complex p_0 plane determines the spectrum of quasiparticles. In vacuum there would be only one pole for positive p_0 at $p_0 = \omega_{\mathbf{p}} \equiv (\mathbf{p}^2 + m^2)^{1/2}$, where m is the renormalised mass. At $T > 0$ there can be several poles, which we will label by an index ^{i}. We refer to the pole that converges to $\omega_{\mathbf{p}}$ in the limit $T \to 0$ as the *screened one-particle state*, and to all other poles as purely collective excitations.

$\Pi^R_{\mathbf{p}}(p_0)$ can be expressed as the sum of a vacuum contribution and a temperature dependent medium correction. The real part of the vacuum contribution contains the usual UV divergence that also appears in vacuum, the temperature dependent part is UV-finite. It is common to impose renormalisation conditions at $T = 0$ to absorb the divergence and define the physical mass [19,20]. We will in the following simply interpret m_ϕ and m_χ as physical masses in vacuum after renormalisation and $\mathrm{Re}\,\Pi^R_{\mathbf{p}}(p_0)$ as the remaining finite piece.[3] Let $\hat{\Omega}^i_{\mathbf{p}}$ be a pole of $\rho_{\mathbf{p}}(p_0)$ with $\Omega^i_{\mathbf{p}} \equiv \mathrm{Re}\,\hat{\Omega}^i_{\mathbf{p}}$ and $\Gamma^i_{\mathbf{p}} \equiv 2\,\mathrm{Im}\,\hat{\Omega}^i_{\mathbf{p}}$. $\Omega^i_{\mathbf{p}}$ and $\Gamma^i_{\mathbf{p}}$ are temperature dependent because $\Pi^R_{\mathbf{p}}(\omega)$ depends on T. In weakly coupled theories one usually observes the hierarchy

$$\Gamma^i_{\mathbf{p}} \ll \Omega^i_{\mathbf{p}}. \quad (7)$$

Due to (7) we can interpret $\Omega^i_{\mathbf{p}}$ as a quasiparticle[4] dispersion relation (or "thermal mass shell") and $\Gamma^i_{\mathbf{p}}$ as its thermal width (or damping rate). Near poles that fulfil (7) the spectral density can be approximated by

$$\rho^{\mathrm{BW}}_{\mathbf{p}}(p_0)\big|_{p_0\simeq\Omega^i_{\mathbf{p}}} \simeq \sum_i 2\mathcal{Z}^i_{\mathbf{p}}\frac{p_0\Gamma^i_{\mathbf{p}}}{(p_0^2 - (\Omega^i_{\mathbf{p}})^2)^2 + (p_0\Gamma^i_{\mathbf{p}})^2} + \rho^{\mathrm{cont}}_{\mathbf{p}}(p_0) \quad (8)$$

Here the residue and width are given by

$$\mathcal{Z}^i_{\mathbf{p}} = \left[1 - \frac{1}{2\Omega^i_{\mathbf{p}}}\frac{\partial\,\mathrm{Re}\,\Pi^R_{\mathbf{p}}(p_0)}{\partial p_0}\right]^{-1}_{p_0=\Omega^i_{\mathbf{p}}},$$

$$\Gamma^i_{\mathbf{p}} = -\mathcal{Z}^i_{\mathbf{p}}\frac{\mathrm{Im}\,\Pi^R_{\mathbf{p}}(\Omega^i_{\mathbf{p}})}{2\Omega^i_{\mathbf{p}}}. \quad (9)$$

In the zero-width limit the it reads

$$\rho^0_{\mathbf{p}}(p_0) = \sum_i \mathcal{Z}^i_{\mathbf{p}}2\pi\,\mathrm{sign}(p_0)\delta\big(p_0^2 - (\Omega^i_{\mathbf{p}})^2\big) + \rho^{\mathrm{cont}}_{\mathbf{p}}(p_0), \quad (10)$$

which can be compared to the free spectral density

$$\rho^{\mathrm{free}}_{\mathbf{p}}(p_0) = 2\pi\,\mathrm{sign}(p_0)\delta\big(p_0^2 - \omega^2_{\mathbf{p}}\big). \quad (11)$$

The dispersion relation in (10) is essentially fixed by $\mathrm{Re}\,\Pi^R_{\mathbf{p}}(p_0)$ via the condition

$$p_0^2 - \mathbf{p}^2 - m^2 - \mathrm{Re}\,\Pi^R_{\mathbf{p}}(p_0) = 0. \quad (12)$$

For this reason the real and imaginary part of the retarded self-energy are often referred to as the dispersive self-energy and dissipative self-energy, respectively.

The dispersion relations $\Omega^i_{\mathbf{p}}$ can have a complicated $\mathbf{p}$-dependence. In limited momentum regimes they can often be approximated by momentum independent "thermal masses". For hard modes $\mathbf{p} \sim T$ it is common to define the *asymptotic mass* M, which depends on T but not on $\mathbf{p}$, by fitting the approximation $(\mathbf{p}^2 + M^2)^{1/2}$ to the full dispersion relation in the regime $\mathbf{p} \gtrsim T$. This approximation is commonly used in transport equations because most particles in a plasma in thermal equilibrium have momenta $\mathbf{p} \sim T$. In this work we are interested in collective

ignore this issue here and consider small excitations around the local minimum at $\phi = \chi = 0$, assuming that (1) is embedded into a bigger framework that stabilises the ground state.

[2] Furthermore, in thermal equilibrium Δ^- and Δ^+ are not independent, but related by the Kubo–Martin–Schwinger relation, which for their Fourier transforms reads $\Delta^+_{\mathbf{p}}(p_0) = \frac{1+2f_B(p_0)}{2}\rho_{\mathbf{p}}(p_0)$. Here f_B is the Bose–Einstein distribution. This is the quantum field theoretical version of the detailed balance relation.

[3] Formally we should use different symbols for the mass parameter appearing in (1) and full self-energy before renormalisation on one hand, and the physical mass and finite part of $\mathrm{Re}\,\Pi^R$ on the other. However, the former do not appear anywhere in the following calculation.

[4] We refer to any pole of a propagator that fulfils (7) as quasiparticle, may it be a screened one-particle state or a collective excitation, and regardless of its spin.