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Perturbative quantum damping of cosmological expansion

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ABSTRACT

Perturbative quantum gravity in the framework of the Schwinger–Keldysh formalism is applied to compute lowest-order corrections to expansion of the Universe described in terms of the spatially flat Friedman–Lemaître–Robertson–Walker solution. The classical metric is approximated by a third degree polynomial perturbation around the Minkowski metric. It is shown that quantum contribution to the classical expansion, though extremely small, damps, i.e. slows down, the expansion (phenomenon of quantum friction).

1. Introduction

The aim of our work is to explicitly show the appearance of quantum generated damping, i.e. slowing down, of the present (accelerating) expansion of the Universe (phenomenon of quantum friction). In principle, quantum corrections to classical gravitational field can be perturbatively calculated in a number of ways. First of all, it is possible to directly derive quantum (one-loop) corrections to classical gravitational field from the graviton vacuum polarization (self-energy), in analogy to the case of the Coulomb potential in QED (see, for example, Berestetskii et al. [1]), the so-called Uehling potential. Such a type of calculations has been already performed for the Schwarzschild solution (Duff [7]), as well as for the spatially flat Friedman–Lemaître–Robertson–Walker (FLRW) metric (Broda [5]). Another approach refers to the energy–momentum tensor, and it has been applied to the Newton potential (see, for example, Bjerrum-Bohr et al. [3], and references therein), to the Reissner–Nordström and the Kerr–Newman solutions (see Donoghue et al. [6]), as well as to the Schwarzschild and the Kerr metrics (see Bjerrum-Bohr et al. [2]). Yet another approach uses the Schwinger–Keldysh (SK) formalism to the case of the Newton potential (see, for example, Park and Woodard [9]). It is argued that only the SK formalism is adequate for time-dependent potentials, hence in particular, in the context of cosmology (see, for example, Weinberg [10], and references therein). Because we aim to perturbatively calculate corrections to the spatially flat FLRW metric, we should use the SK formalism, as that is exactly the case (time-dependence of gravitational field) the SK approach has been devised for.

The corrections we calculate are a quantum response to the spatially flat FLRW solution which is described by a small perturbation around the Minkowski metric. For definiteness, we confine ourselves to the classical perturbation given by a third degree polynomial. The final result is expressed in terms of the present time quantum correction q_0^Q to the classical deceleration parameter q_0^C. On the premises assumed, it appears that q_0^Q is positive, though obviously, it is extremely small.

2. Quantum damping

Our starting point is a general spatially flat FLRW metric

$$ds^2 \equiv g_{\mu\nu}\, dx^\mu\, dx^\nu = -dt^2 + a^2(t)\, d\boldsymbol{x}^2, \quad \mu, \nu = 0, 1, 2, 3, \tag{1}$$

with the cosmological scale factor $a(t)$. To satisfy the condition of weakness of the perturbative gravitational field $h_{\mu\nu}$ near our reference time $t = t_0$ (where t_0 could be the age of the Universe—the present moment) in the expansion

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x), \tag{2}$$

the metric should be normalized in such a way that it is exactly Minkowskian for $t = t_0$, i.e.

$$a^2(t) = 1 + h(t), \qquad h(t_0) = 0. \tag{3}$$

(Let us note the analogy to the Newton potential ($\sim 1/r$), where the "reference radius" is in spatial infinity, i.e. $r_0 = +\infty$.) Then, in the block diagonal form,

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$$h_{\mu\nu}(t,\boldsymbol{x}) = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij}h(t) \end{pmatrix}, \quad i,j=1,2,3. \tag{4}$$

To obtain quantum corrections to the classical gravitational field $h^{\mathrm{C}}_{\mu\nu}(x)$, we shall use the one-loop effective field equation derived by Park and Woodard [9],

$$\mathcal{D}^{\mu\nu\varrho\sigma}h^{\mathrm{Q}}_{\varrho\sigma}(t,\boldsymbol{x}) = \frac{\kappa^2}{10\,240\pi^3}\mathrm{D}^{\mu\nu\varrho\sigma}\partial^4\int_0^t dt'\int d^3x'\,\theta(\Delta t-\Delta r) \times \left[\ln\left(-\mu^2\Delta x^2\right)-1\right]h^{\mathrm{C}}_{\varrho\sigma}\left(t',\boldsymbol{x}'\right), \tag{5}$$

where $\Delta t \equiv t-t'$, $\Delta r \equiv |\boldsymbol{x}-\boldsymbol{x}'|$, $\Delta x^2 \equiv -(\Delta t)^2+(\Delta r)^2$, and the mass scale μ is coming from the UV renormalization procedure (see Ford and Woodard [8]). Here $\kappa^2 = 16\pi G_N$, where G_N is the Newton gravitational constant. The operator $\mathcal{D}$ (the Lichnerowicz operator in the flat background) is of the form

$$\mathcal{D}^{\mu\nu\varrho\sigma} = \frac{1}{2}\left(\eta^{\mu\nu}\eta^{\varrho\sigma}\partial^2 - \partial^\mu\partial^\nu\eta^{\varrho\sigma} - \eta^{\mu\nu}\partial^\varrho\partial^\sigma - \eta^{\mu(\varrho}\eta^{\sigma)\nu}\partial^2 + 2\partial^{(\mu}\eta^{\nu)(\varrho}\partial^{\sigma)}\right),$$

and for the minimally coupled massless scalar field

$$\mathrm{D}^{\mu\nu\varrho\sigma} = \Pi^{\mu\nu}\Pi^{\varrho\sigma} + \frac{1}{3}\Pi^{\mu(\varrho}\Pi^{\sigma)\nu} \tag{6}$$

with

$$\Pi^{\mu\nu} \equiv \eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu.$$

For conformally coupled fields we have $\widetilde{\mathrm{D}}$ instead of D, where

$$\widetilde{\mathrm{D}}^{\mu\nu\varrho\sigma} \equiv -\frac{1}{9}\Pi^{\mu\nu}\Pi^{\varrho\sigma} + \frac{1}{3}\Pi^{\mu(\varrho}\Pi^{\sigma)\nu}.$$

Since the metric depends only on time, we can explicitly perform the spatial integration with respect to $\boldsymbol{x}'$ in (5), obtaining the integral kernel (time propagator)

$$\mathrm{K}(\Delta t) \equiv 4\pi\int_0^{\Delta t} dr\,r^2\left\{\ln\left[\mu^2\left((\Delta t)^2-r^2\right)\right]-1\right\} = \frac{4\pi}{3}(\Delta t)^3\left[\ln\left(4\mu^2\Delta t^2\right)-\frac{11}{3}\right]. \tag{7}$$

For the time-dependent metric of the form

$$\begin{pmatrix} f(t) & \\ & \delta_{ij}h(t) \end{pmatrix},$$

the action of the operators $\mathcal{D}$, D and $\widetilde{\mathrm{D}}$ is given by

$$\mathcal{D}\begin{pmatrix} f(t) & \\ & \delta_{ij}h(t) \end{pmatrix} = \begin{pmatrix} 0 & \\ & -\delta_{ij}\frac{d^2}{dt^2}h(t) \end{pmatrix}, \tag{8}$$

$$\mathrm{D}\begin{pmatrix} f(t) & 0 \\ 0 & \delta_{ij}h(t) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{10}{3}\delta_{ij}\frac{d^4}{dt^4}h(t) \end{pmatrix}, \tag{9}$$

and

$$\widetilde{\mathrm{D}}\begin{pmatrix} f(t) & 0 \\ 0 & \delta_{ij}h(t) \end{pmatrix} = 0, \tag{10}$$

respectively. There are no mixing of diagonal and non-diagonal terms, and the empty blocks mean expressions which can be non-zero, but they are inessential in our further analysis. Thus, (5) assumes the simple form

$$\frac{d^2}{dt^2}h^{\mathrm{Q}}(t) = -\frac{\kappa^2}{3072\pi^3}\frac{d^8}{dt^8}\left(\mathrm{K}\star h^{\mathrm{C}}\right)(t), \tag{11}$$

where the integral kernel K is given by (7), and the convolution "$\star$" is standardly defined by

$$(\mathrm{K}\star\mathrm{F})(t) \equiv \int_0^t \mathrm{K}\left(t-t'\right)\mathrm{F}\left(t'\right)dt' = \int_0^t \mathrm{K}\left(t'\right)\mathrm{F}\left(t-t'\right)dt'. \tag{12}$$

One should note that due to the diagonal form of (4) and (8)–(10), no non-diagonal terms of the metric enter (11).

Since the upper limit of integration in (12) depends on t, the derivative of the convolution with respect to t is expressed by

$$\frac{d^n}{dt^n}(\mathrm{K}\star\mathrm{F})(t) = \left(\frac{d^n}{dt^n}\mathrm{K}\star\mathrm{F}\right)(t) + \sum_{k=1}^{n}\frac{d^{(n-k)}}{dt^{(n-k)}}\mathrm{K}(0)\frac{d^{(k-1)}}{dt^{(k-1)}}\mathrm{F}(t). \tag{13}$$

Using symmetry between K and F, Eq. (12), it is possible to distribute differentiation in (13) in several different ways. For practical purposes, further analysis, the most convenient form of the eighth derivative is the "symmetric" one, i.e.

$$\frac{d^8}{dt^8}\left(\mathrm{K}\star h^{\mathrm{C}}\right)(t) = \left(\frac{d^4}{dt^4}\mathrm{K}\star\frac{d^4}{dt^4}h^{\mathrm{C}}\right)(t) + \sum_{k=1}^{4}\left[\frac{d^{(4-k)}}{dt^{(4-k)}}\mathrm{K}(0)\frac{d^{(k+3)}}{dt^{(k+3)}}h^{\mathrm{C}}(t) + \frac{d^{(4-k)}}{dt^{(4-k)}}h^{\mathrm{C}}(0)\frac{d^{(k+3)}}{dt^{(k+3)}}\mathrm{K}(t)\right]. \tag{14}$$

To prevent the appearance of the mass scale μ, as well as "classical" divergences in the convolution, which could possibly come from singularities in the kernel (time propagator) K, we assume the following third degree polynomial form of the classical metric

$$h^{\mathrm{C}}(\tau) = h_0 + h_1\tau + h_2\tau^2 + h_3\tau^3. \tag{15}$$

Henceforth, for simplicity, instead of t we use the dimensionless unit of time, $\tau \equiv t/t_0$.

The well-defined form of Eq. (16) proofs that (15) has been properly selected. In fact, our choice is unique. First of all, let us observe that the UV renormalized equation of motion (11) is well-defined, at least by classical standards. This means that it may happen for some $h^{\mathrm{C}}(\tau)$ that Eq. (11) is not integrable for the kernel (7), but non-integrabilities may appear also in standard classical field theory, e.g. self-energy of a point particle in classical electrodynamics. Hence, in our calculations, possible infinities are considered as "classical". Their presence depends on the form of h^{C}, and it could be interpreted, as usually in classical field theory, as inapplicability of the approach in such a type of problem. Therefore, following that point of view, we should avoid contributions from

$$\left.\frac{d^k}{dt^k}\mathrm{K}(t)\right|_{t=0}, \quad \text{for } k>2,$$

because they generate singularities in Eq. (14), due to the singular form of the kernel.

Another issue concerns the mass scale μ present in (7), which results from renormalization procedure. There are the two possibilities. One can choose some "natural" mass scale μ, or one can confine oneself to μ-independent cases. The second possibility, if available, is preferable because it gives unambiguous results. For example, let us consider quantum corrections to black-hole