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Transverse invariant higher-spin fields

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ABSTRACT

It is shown that a symmetric massless bosonic higher-spin field can be described by a traceless tensor field with reduced (transverse) gauge invariance. The Hamiltonian analysis of the transverse gauge invariant higher-spin models is used to control a number of degrees of freedom.

1. Introduction

It was shown recently in [1,2] that a free massless spin two field (i.e., linearized gravity) can be consistently described by a traceless rank-2 tensor field with transverse gauge symmetry that corresponds to linearized volume-preserving diffeomorphisms. We extend this result to massless fields of arbitrary spin by showing that a spin-s symmetric massless field can be described by a rank-s traceless symmetric tensor. This formulation is in some sense opposite to the approach developed in [3–5] where a massless field is described by a traceful tensor. Recall that the standard Fronsdal's formulation of a spin-s massless field operates with a rank-s double traceless tensor [6]. For recent reviews on higher-spin (HS) gauge theories see [7].

Although, like in the case of gravity, the obtained model is a gauge fixed version of the original Fronsdal model [6] the equivalence is not completely trivial. Actually, the standard counting of degrees of freedom is that each gauge parameter in the gauge transformations with first order derivatives kills two degrees of freedom [9]. Therefore one can expect that the invariance under reduced gauge symmetry may be not sufficient to compensate all extra degrees of freedom. As we show this is not the case. The reason is that the remaining gauge symmetry parameters satisfy the differential transversality conditions $\partial^\nu \xi_{\nu\mu_2\ldots\mu_{s-1}} = 0$.

Generally, as explained in this Letter, a partial gauge fixing at the Lagrangian level can give rise to a model which, if treated independently of the original gauge model, may differ from the latter. In particular, the Hamiltonian interpretation of the gauge fixed Lagrangian model may differ from that of the original model. This can happen in the case where the gauges and constraints on gauge parameters are differential. For example, as shown in Section 5, this does happen in electrodynamics in the temporary gauge. Since the transversality condition on the gauge parameter is also of this type, a more careful analysis of the counting of the number of degrees of freedom in the model under consideration is needed. The Hamiltonian analysis of Section 5 shows that the transverse gauge invariant HS model has as many degrees of freedom as the original Fronsdal model in the topologically trivial situation.

Note, that the original Lagrangian and field/gauge transformations content for a massless field of an arbitrary spin were derived by Fronsdal in [6] by taking the zero rest mass limit $m^2 \to 0$ in the Lagrangian of Singh and Hagen for a massive HS field of [8]. To the best of our knowledge, it has not been analyzed in the literature what is a minimal field content appropriate for the description a massless HS field. The proposed formulation operates in terms of an irreducible Lorentz tensor field, thus being minimal. It is equivalent to the Fronsdal's one in the topologically trivial situation but may differ otherwise.

Also let us note that since the minimal formulation has a relaxed gauge symmetry compared to that of the Fronsdal's formulation, it may in principle have more freedom at the interaction level, i.e., all interactions which can be introduced for the Fronsdal's theory are automatically recovered in its gauge fixed version.

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However, as the gauge symmetry is weaker in the proposed formulation, some new types of interactions can in principle be expected. Note that the recovery of usual gravitational interactions in the case of spin-2 was shown in the model of [2] with the diffeomorphism symmetry of Einstein theory relaxed to volume preserving diffeomorphisms.

The layout of the rest of the Letter is as follows. In Section 2 we recall the standard description of massive and massless fields of arbitrary spin. In Section 3, transverse and Weyl invariant Lagrangian is constructed and a generating action is given. The equivalence of transverse and Weyl invariant Lagrangian to the Fronsdal's Lagrangian is checked in Section 4. Hamiltonian analysis and examples are given in Section 5.

2. Free massless higher-spin fields

A spin-s bosonic totally symmetric massive field in Minkowski space can be described on shell [10] by a totally symmetric tensor field $\varphi_{\mu_1\ldots\mu_s}$[1] that satisfies the conditions

$$(\Box+m^2)\varphi_{\mu_1\ldots\mu_s}=0,\qquad \partial^\nu\varphi_{\nu\mu_2\ldots\mu_s}=0,\qquad \varphi^\nu{}_{\nu\mu_3\ldots\mu_s}=0. \tag{2.1}$$

These form the complete set of local Poincaré-invariant conditions on $\varphi_{\mu_1\ldots\mu_s}$. In the massless case $m^2=0$ a gauge invariance with an on-shell traceless rank-$(s-1)$ tensor gauge parameter reduces further the number of physical degrees of freedom.

As pointed out by Fierz and Pauli in [11], for (2.1) to be derivable from a Lagrangian a set of auxiliary fields has to be added for $s>1$ (in the case of spin two considered by Fierz and Pauli this is a scalar auxiliary field φ, which together with a traceless $\varphi_{\mu_1\mu_2}$ forms a traceful field $\phi_{\mu_1\mu_2}=\varphi_{\mu_1\mu_2}+\eta_{\mu_1\mu_2}\varphi$). Auxiliary fields are zero on shell, thus carrying no physical degrees of freedom. For totally symmetric massive fields of integer spins, the Lagrangian formulation with a minimal set of auxiliary fields was worked out by Singh and Hagen in [8]. For a spin-s field they introduced a set of auxiliary fields, which consists of symmetric traceless tensors of ranks $s-2, s-3, \ldots, 0$. An elegant gauge invariant (Stueckelberg) formulation was proposed by Zinoviev in [12]. (For alternative approaches to massive fields see also [13–15] and references therein.)

The Lagrangian of a spin-s massless field can be obtained [6] in the limit $m^2\to 0$. The auxiliary fields of ranks from 0 to $(s-3)$ decouple while the residual rank-$(s-2)$ traceless auxiliary field $\varphi_{\mu_1\ldots\mu_{s-2}}$ and the physical rank-s traceless field $\varphi_{\mu_1\ldots\mu_s}$ form the symmetric field $\phi_{\mu_1\ldots\mu_s}=\varphi_{\mu_1\ldots\mu_s}+\eta_{(\mu_1\mu_2}\varphi_{\mu_3\ldots\mu_{s-2})}$ that satisfies the double tracelessness condition

$$\eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\phi_{\mu_1\ldots\mu_s}=0, \tag{2.2}$$

which makes sense for $s\geqslant 4$. The resulting Lagrangian possesses gauge invariance with a traceless rank-$(s-1)$ gauge parameter $\xi_{\mu_1\ldots\mu_{s-1}}$,

$$\delta\phi_{\mu_1\ldots\mu_s}=s\partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)},\qquad \xi^\nu{}_{\nu\mu_3\ldots\mu_{s-1}}=0. \tag{2.3}$$

In the spin two case of linearized gravity, the gauge law (2.3) corresponds to linearized diffeomorphisms.

Let us write down a most general bilinear action and Lagrangian (modulo total derivatives) of a double traceless field with at most two derivatives as

$$\mathcal{L}=(-)^s\sum_{\alpha=a,b,c,f,g}\mathcal{L}_\alpha,\qquad S=\int d^dx\,\mathcal{L}, \tag{2.4}$$

where

$$\begin{aligned}
\mathcal{L}_a&=\frac{a}{2}\partial_\nu\phi_{\mu_1\ldots\mu_s}\partial^\nu\phi^{\mu_1\ldots\mu_s},\\
\mathcal{L}_b&=-\frac{bs(s-1)}{4}\partial_\nu\phi^\rho{}_{\rho\mu_3\ldots\mu_s}\partial^\nu\phi_\lambda{}^{\lambda\mu_3\ldots\mu_s},\\
\mathcal{L}_c&=-\frac{cs}{2}\partial^\nu\phi_{\nu\mu_2\ldots\mu_s}\partial_\rho\phi^{\rho\mu_2\ldots\mu_s},\\
\mathcal{L}_f&=\frac{fs(s-1)}{2}\partial_\nu\phi^\rho{}_{\rho\mu_3\ldots\mu_s}\partial_\lambda\phi^{\lambda\nu\mu_3\ldots\mu_s},\\
\mathcal{L}_g&=-\frac{gs(s-1)(s-2)}{8}\partial^\nu\phi^\rho{}_{\rho\nu\mu_4\ldots\mu_s}\partial_\lambda\phi_\sigma{}^{\sigma\lambda\mu_4\ldots\mu_s}
\end{aligned} \tag{2.5}$$

with arbitrary coefficients a, b, c, f, g. For $\mathcal{L}$ to describe a spin-s field, the coefficient a has to be nonzero (so, we set $a=1$).

The variation of (2.4) is

$$\delta\mathcal{L}=\left(G_{\mu_1\ldots\mu_s}-\frac{s(s-1)}{2(\Upsilon-2)}\eta_{(\mu_1\mu_2}G^\rho{}_{\rho\mu_3\ldots\mu_s)}\right)\delta\phi^{\mu_1\ldots\mu_s}, \tag{2.6}$$

where $\Upsilon=d+2s-4$ and

$$\begin{aligned}
G_{\mu_1\ldots\mu_s}&=\Box\phi_{\mu(s)}-b\frac{s(s-1)}{2}\eta_{\mu\mu}\Box\phi^\lambda{}_{\lambda\mu(s-2)}-cs\partial_\mu\partial^\nu\phi_{\nu\mu(s-1)}\\
&\quad+f\frac{s(s-1)}{2}\left(\eta_{\mu\mu}\partial^\nu\partial^\lambda\phi_{\nu\lambda\mu(s-2)}+\partial_\mu\partial_\mu\phi^\lambda{}_{\lambda\mu(s-2)}\right)\\
&\quad-g\frac{s(s-1)(s-2)}{4}\eta_{\mu\mu}\partial_\mu\partial^\nu\phi^\lambda{}_{\lambda\nu\mu(s-3)}.
\end{aligned} \tag{2.7}$$

The requirement that the action is invariant under (2.3) fixes the coefficients $a=b=c=f=g$ [16].

3. Transverse and Weyl invariant massless higher-spin fields

Let us consider a weaker condition on the action imposed by the reduced gauge symmetry (2.3) with the transverse gauge parameter $\xi_{\mu_1\ldots\mu_{s-1}}$

$$\begin{aligned}
&\delta\phi_{\mu_1\ldots\mu_s}=s\partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)},\qquad \partial^\nu\xi_{\nu\mu_2\ldots\mu_{s-1}}=0,\\
&\xi^\nu{}_{\nu\mu_3\ldots\mu_{s-1}}=0.
\end{aligned} \tag{3.1}$$

The invariance of action (2.4) under (3.1) fixes only the ratio $a/c=1$ while the rest of the coefficients remains free. This ambiguity can be used to look for another symmetry to kill extra degrees of freedom. Taking into account the double tracelessness condition (2.2), a use of rank-$(s-2)$ symmetric traceless gauge parameter $\zeta_{\mu_1\ldots\mu_{s-2}}$ is a natural option

$$\delta\phi_{\mu_1\ldots\mu_s}=\frac{s(s-1)}{2}\eta_{(\mu_1\mu_2}\zeta_{\mu_3\ldots\mu_s)},\qquad \zeta^\nu{}_{\nu\mu_3\ldots\mu_{s-2}}=0. \tag{3.2}$$

The requirement for (2.4) to be invariant under the additional (Weyl) symmetry (3.2) fixes the rest of the coefficients

$$b=\frac{\Upsilon+2}{\Upsilon^2},\qquad f=\frac{2}{\Upsilon},\qquad g=\frac{-2(\Upsilon-4)}{\Upsilon^2}. \tag{3.3}$$

Note that, not too surprisingly, the resulting Lagrangian (2.4) can be obtained from the Fronsdal's Lagrangian (i.e., that with $a=b=c=f=g=1$) via the substitution

$$\begin{aligned}
&\tilde\phi_{\mu_1\ldots\mu_s}=\phi_{\mu_1\ldots\mu_s}-\frac{1}{\Upsilon}\frac{s(s-1)}{2}\eta_{(\mu_1\mu_2}\phi^\nu{}_{\nu\mu_3\ldots\mu_s)},\\
&\tilde\phi^\nu{}_{\nu\mu_3\ldots\mu_s}=0.
\end{aligned} \tag{3.4}$$

There is a generating action S^{gen} that gives rise both to the Fronsdal and to the Weyl invariant actions in particular gauges. S^{gen} results from the Fronsdal action by introducing a traceless Stueckelberg field $\chi_{\mu_1\ldots\mu_{s-2}}$ of rank-$(s-2)$ via the substitution

$$\phi_{\mu_1\ldots\mu_s}\to\phi_{\mu_1\ldots\mu_s}+\frac{s(s-1)}{2}\eta_{(\mu_1\mu_2}\chi_{\mu_3\ldots\mu_s)}, \tag{3.5}$$

[1] Greek indices $\mu,\nu,\lambda,\rho=0,\ldots,d-1$ are vector indices of d-dimensional Lorentz algebra $o(d-1,1)$. $\partial_\mu\equiv\frac{\partial}{\partial x^\mu}$, $\Box\equiv\partial^\nu\partial_\nu$ and indices are raised and lowered by mostly minus invariant tensor $\eta_{\mu\nu}$ of $o(d-1,1)$. A group of indices to be symmetrized is denoted by placing them in brackets or, shortly, by the same letter. For example, $\partial_\mu\phi_\mu\equiv\partial_{(\mu_1}\phi_{\mu_2)}\equiv\frac{1}{2}(\partial_{\mu_1}\phi_{\mu_2}+\partial_{\mu_2}\phi_{\mu_1})$.