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Route to Lambda in conformally coupled phantom cosmology

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Abstract

In this Letter we investigate acceleration in the flat cosmological model with a conformally coupled phantom field and we show that acceleration is its generic feature. We reduce the dynamics of the model to a 3-dimensional dynamical system and analyze it on a invariant 2-dimensional submanifold. Then the concordance FRW model with the cosmological constant Λ is a global attractor situated on a 2-dimensional invariant space. We also study the behaviour near this attractor, which can be approximated by the dynamics of the linearized part of the system. We demonstrate that trajectories of the conformally coupled phantom scalar field with a simple quadratic potential crosses the cosmological constant barrier infinitely many times in the phase space. The universal behaviour of the scalar field and its potential is also calculated. We conclude that the phantom scalar field conformally coupled to gravity gives a natural dynamical mechanism of concentration of the equation of state coefficient around the magical value $w_{\text{eff}} = -1$. We demonstrate route to Lambda through the infinite times crossing the $w_{\text{eff}} = -1$ phantom divide.

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At present the scalar fields play a crucial role in modern cosmology. In an inflationary scenario they generate an exponential rate of evolution of the universe as well as density fluctuations due to vacuum energy. The Lagrangian for a phantom scalar field on the background of the Friedmann–Robertson–Walker (FRW) universe is assumed in the form

$$\mathcal{L}_\psi = \frac{1}{2}\left[g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + \xi R\psi^2 - 2U(\psi)\right], \tag{1}$$

where $g^{\mu\nu}$ is the metric of the spacetime manifold, $\psi = \psi(t)$, t is the cosmological time, $R = R(g)$ is the Ricci scalar for the spacetime metric g, ξ is a coupling constant which assumes zero for a scalar field minimally coupled to gravity and $1/6$ for a conformally coupled scalar field, $U(\psi)$ is a potential of the scalar field.

The minimally coupled slowly evolving scalar fields with a potential function $U(\psi)$ are good candidates for a description of dark energy. In this model, called quintessence [1,2], the energy density and pressure from the scalar field are $\rho_\psi = -1/2\dot{\psi}^2 + U(\psi)$, $p_\psi = -1/2\dot{\psi}^2 - U(\psi)$. From recent studies of observational constraints we obtain that $w_\psi \equiv p_\psi/\rho_\psi < -0.55$ [3]. This model has been also extended to the case of a complex scalar field [4,5].

Observations of distant supernovae support the cosmological constant term which corresponds to the case $\dot{\psi} \simeq 0$. Then we obtain that $w_\psi = -1$. But there emerge two problems in this context. Namely, the fine tuning and the cosmic coincidence problems. The first problem comes from the quantum field theory where the vacuum expectation value is of 123 orders of magnitude larger than the observed value of 10^{-47} GeV^4. The lack of a fundamental mechanism which sets the cosmological constant almost zero is

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called the cosmological constant problem. The second problem called "cosmic conundrum" is a question why the energy densities of both dark energy and dark matter are nearly equal at the present epoch.

One of the solutions to this problem offers the idea of quintessence, which is a version of the time varying cosmological constant conception. Quintessence solves the first problem through the decaying Λ term from the beginning of the Universe to a small value observed at the present epoch. Also the ratio of energy density of this field to the matter density increases slowly during the expansion of the Universe because the specific feature of this model is the variation of the coefficient of the equation of state with respect to time. The quintessence models [2,6] describe the dark energy with the time varying equation of state for which $w_X > -1$, but recently quintessence models have been extended to the phantom quintessence models with $w_X < -1$. In this class of models the weak energy condition is violated and such a theoretical possibility is realized by a scalar field with a switched sign in the kinetic term $\dot{\psi}^2 \to -\dot{\psi}^2$ [7–9]. From theoretical point of view it is necessary to explore different evolutional scenarios for dark energy which provide a simple and natural transition to $w_X = -1$. The methods of dynamical systems with notion of attractor (a limit set with an open inset) offers the possibility of description of transition trajectories to the regime with $w_X = -1$. Moreover they demonstrate whether this mechanism is generic.

Inflation and quintessence with non-minimal coupling constant are studied in the context of formulation of necessary conditions for the acceleration of the universe [10] (see also [11,12]). We can find two important arguments which favour the choice of conformal coupling over $\xi \neq 1/6$. The first, equation for the massless scalar field is conformally invariant [13,14]. The second argument is that if the scalar field satisfy Klein–Gordon equation in the curved space then ψ does not violate the equivalence principle, and ξ is forced to assume the value $1/6$ [15].

While recent astronomical observations give support that the equation of state parameter for dark energy is close to constant value -1 they do not give a "corridor" around this value. Moreover, Alam et al. [16] pointed out that evolving state parameter is favoured over constant $w_X = -1$. The first step in the direction of description of the dynamics of the dark energy seems to be investigation of the system with evolving dark energy in the close neighbourhood of the value $w_X = -1$. For this aim we linearize dynamical system at this critical point and then describe the system in a good approximation (following the Hartman–Grobman theorem [17]) by its linearized part.

Other dark energy models like the Chaplygin gas model [18–20], [9, and references therein] and the model with tachyonic matter can also be interpreted in terms of a scalar field with some form of a potential function.

Recent applications of the Bayesian framework [21–25] of model selection to the broad class of cosmological models with acceleration indicate that a posteriori probability for the ΛCDM model is 96%. Therefore the explanation why the current universe is such close to the ΛCDM model seems to be a major challenge for modern theoretical cosmology.

In this Letter we present the simplest mechanism of concentration around $w_X = -1$ basing on the influence of a single scalar field conformally coupled to gravity acting in the radiation epoch. Phantom cosmology non-minimally coupled to the Ricci scalar was explored in the context of superquintesence ($w_X < -1$) by Faraoni [26,27] and there was pointed out that the superacceleration regime can be achieved by the conformally coupled scalar field in contrast to the minimally coupled scalar field.

Let us consider the flat FRW model which contains a negative kinetic scalar field conformally coupled to gravity ($\xi = 1/6$) (phantom) with the potential function $U(\psi)$. For the simplicity of presentation we assume $U(\psi) \propto \psi^2$. In this model the phantom scalar field is coupled to gravity via the term $\xi R\psi^2$. We consider massive scalar fields (for recent discussion of cosmological implications of massive and massless scalar fields see [28]). The dynamics of a non-minimally coupled scalar field for some self-interacting potential $U(\psi)$ and for an arbitrary ξ is equivalent to the action of the phantom scalar field (which behaves like a perfect fluid) with energy density ρ_ψ and pressure p_ψ [29],

$$\rho_\psi = -\frac{1}{2}\dot{\psi}^2 + U(\psi) - 3\xi H^2\psi^2 - 3\xi H\big(\psi^2\big)^{\cdot}, \tag{2}$$

$$p_\psi = -\frac{1}{2}\dot{\psi}^2 - U(\psi) + \xi\big[2H\big(\psi^2\big)^{\cdot} + \big(\psi^2\big)^{\cdot\cdot}\big] + \xi\big[2\dot{H} + 3H^2\big]\psi^2, \tag{3}$$

where the conservation condition $\dot{\rho}_\psi = -3H(\rho_\psi + p_\psi)$ gives rise to the equation of motion for the field

$$\ddot{\psi} + 3H\dot{\psi} + \xi R\psi^2 - U'(\psi) = 0, \tag{4}$$

where $R = 6(\dot{H} + 2H^2)$ is the Ricci scalar.

Let us assume that both the homogeneous scalar field $\psi(t)$ and the potential $U(\psi)$ depend on time through the scale factor, i.e.

$$\psi(t) = \psi\big(a(t)\big), \qquad U(\psi) = U\big(\psi(a)\big); \tag{5}$$

then due to this simplified assumption the coefficient of the equation of state w_ψ is parameterized by the scale factor only

$$w_\psi = w_\psi(a), \qquad p_\psi = w_\psi(a)\rho_\psi(a), \tag{6}$$

and

$$w_\psi = \frac{-\frac{1}{2}\psi'^2H^2a^2 - U(\psi) + \xi[2(\psi^2)'H^2a + (\psi^2)^{\cdot\cdot}] + \xi[\dot{H} + 3H^2]\psi^2}{-\frac{1}{2}\psi'^2H^2a^2 + U(\psi) - 3\xi H^2\psi^2 - 3\xi(\psi^2)'H^2a} \tag{7}$$