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First law of thermodynamics and Friedmann-like equations in braneworld cosmology

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Abstract

We derive the Friedmann-like equations in braneworld cosmology by imposing the first law of thermodynamics and Bekenstein's area-entropy formula on the apparent horizon of a Friedmann–Robertson–Walker universe in both Randall–Sundrum II gravity and Dvali–Gabadadze–Porrati gravity models. Israel's boundary condition plays an important role in our calculations in both cases, besides the first law of thermodynamics and Bekenstein's area-entropy formula. The results indicate that thermodynamics on the brane world knows the behaviors of gravity.

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1. Introduction

The relations between thermodynamics of space–time and the nature of gravity is one of the most intriguing topics in theoretical physics. The four laws of black hole thermodynamics were first derived from the classical Einstein equation [1]. With the discovery of Hawking radiation of black holes [2], it became clear that the analogy is actually an identity [3]. On the other hand, the Einstein equation is derived by Jacobson from the proportionality of entropy and horizon area together with the first law of thermodynamics $\delta Q = T\,dS$ [4]. Verlinde found that the Friedmann equation in a radiation dominated Friedmann–Robertson–Walker (FRW) universe can be written in an analogous form of the Cardy–Verlinde formula, an entropy formula for a conformal field theory [5]. The above observations imply that thermodynamics of space–time and the Einstein equation are closely related [6]. In Ref. [7], Cai and Kim derived the Friedmann equations of FRW universe with any spatial curvature by applying the first law of thermodynamics to the apparent horizon and assuming the geometric entropy is given by a quarter of the apparent horizon area.

The main interest of this Letter relays on the relations between the first law of thermodynamics and the Friedmann-like equation in braneworld cosmology. In the study of three-brane cosmological models, an unusual law of cosmological expansion on the brane has been reported (for incomplete references see [8–12]). According to this law, the energy density of matter on the brane quadratically enters the right-hand of the new Friedmann equation for the braneworld cosmology, in contrast with the standard cosmology, where the Friedmann equation depends linearly on the energy density of matter. In this Letter, we shall derive the Friedmann-like equation in the braneworld cosmology from the first law of thermodynamics.

This Letter is organized as follows. In Section 2, we discuss the relations between thermodynamics and gravity in Randall–Sundrum II (RSII) model [13]. In Section 3, we derive the Friedmann-like equation in Dvali–Gabadadze–Porrati (DGP) model [14] by applying the first law of thermodynamics to the apparent horizon together with Israel's boundary condition. We present our conclusions in Section 4.

2. FRW universe in Randall–Sundrum II gravity

We consider a D-brane located on the boundary of an $(n+1)$-dimensional anti-de Sitter space–time, as our visible

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universe with ordinary matter being trapped on this brane by string theory effects. We will derive the braneworld cosmological evolution equations from the first law of thermodynamics. At each point of on the brane, we define a space–time unit normal, $N_A = N_A(x)$, to the surface that satisfies $g^{AB}N_A N_B = 1$. g^{AB} is the bulk metric and the indices A, B run over all the bulk coordinates. The bulk metric induces a metric on the brane,

$$g_{\mu\nu} = g_{AB} - N_{AB}. \tag{1}$$

The Einstein equation on the 3-brane world has been derived in [15]. The Einstein equation on this $(n-1)$-brane is given by [16,17] (we use the notations in [16]),

$$^{(n)}G_{\mu\nu} = -\Lambda_n g_{\mu\nu} + 8\pi G_n T_{\mu\nu} + \kappa_{n+1}^4 \Pi_{\mu\nu} - E_{\mu\nu}, \tag{2}$$

where

$$\Lambda_n = \kappa_{n+1}^2\left[\frac{n-2}{n}\Lambda_{n+1} + \frac{(n-2)}{8(n-1)}\kappa_{n+1}^2\lambda^2\right], \tag{3}$$

$$G_n = \frac{n-2}{32\pi(n-1)}\lambda\kappa_{n+1}^4, \tag{4}$$

$$\Pi_{\mu\nu} = -\frac{1}{4}T_{\mu\alpha}T_\nu^\alpha + \frac{1}{4(n-1)}TT_{\mu\nu} + \frac{1}{8}g_{\mu\nu}T_{\alpha\beta}T^{\alpha\beta} - \frac{1}{8(n-1)}T^2 g_{\mu\nu}, \tag{5}$$

where $\kappa_{n+1}^2 = 8\pi G_{n+1}$, Λ_n is the brane cosmological constant, λ is the brane tension, G_n is the Newton's constant in n-dimensions, $T_{\mu\nu}$ is the energy–momentum tensor of matter in the brane world, and $E_{\mu\nu}$ is the Weyl tensor, which is vanishing in pure anti-de Sitter space–time. We can see from (2) that the n-dimensional effective equations of motion, being given entirely in terms of quantities defined on the brane, is independent of the evolution of the bulk space–time. Different from the standard Einstein's equation in general relativity, the energy–momentum tensor in the right-hand of (2) is modified due to the presence of extrinsic curvature when project $(n+1)$-dimensional bulk quantities to the brane. We will not solve (2) in the rest of our work, but just utilize the modified energy–momentum tensor in the right-hand of (2) together with the first law of thermodynamics to derive cosmological evolution equation on the brane.

In the following calculation, we assume the cosmological constant Λ_n vanishes, since we are mainly concern with the dynamical aspects of the braneworld cosmology while Λ_n is a constant and does not vary with time. The induced metric on the brane is the FRW metric with the form,

$$ds_n^2 = -d\tau^2 + a^2(\tau)\left(\frac{dr^2}{1-kr^2} + r^2\,d\Omega_{n-2}^2\right), \tag{6}$$

where $d\Omega_{n-2}^2$ is the metric on an $(n-2)$-dimensional Euclidean unit space of constant curvature, $k = 1, 0, -1$ which corresponds to the unit sphere, plane, and hyperboloid respectively. The metric (6) can be rewritten as [18]

$$ds^2 = h_{ab}\,dx^a\,dx^b + \tilde{r}^2\,d\Omega_{n-2}^2, \tag{7}$$

where $\tilde{r} = a(\tau)r$ and $x^0 = \tau$, $x^1 = r$. The 2-dimensional metric $h_{ab} = \mathrm{diag}(-1, \frac{a^2(\tau)}{1-kr^2})$. Since the apparent horizon satisfies the equation $h^{ab}\partial_a\tilde{r}\partial_b\tilde{r} = 0$, we obtain the radius of the apparent horizon,

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}, \tag{8}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. For a dynamical space–time, the apparent horizon is regarded as the horizon satisfying the Bekenstein area-entropy formula [19]. A unified law of black hole dynamics and relativistic thermodynamics is derived in spherically symmetric general relativity, where the gradient of the active gravitational energy E determined by the Einstein's equation is divided into *energy-supply* and *work* terms [19]. Assume the matter on the brane is given by a homogeneous perfect fluid of density ρ and pressure p, so that

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + pg_{\mu\nu}. \tag{9}$$

Substituting the tensor back to Eq. (2) and keep in mind that Λ_n and $E_{\mu\nu}$ are vanishing, we find that,

$$\begin{aligned}\tilde{T}_\nu^\mu &= T_\nu^\mu + \frac{\kappa_{n+1}^4}{\kappa_n^2}\Pi_\nu^\mu \\ &= \mathrm{diag}\left(-\rho - \frac{n-2}{8(n-1)}\frac{\kappa_{n+1}^4}{\kappa_n^2}\rho^2,\right. \\ &\qquad \left. p + \frac{n-2}{8(n-1)}\frac{\kappa_{n+1}^4}{\kappa_n^2}\rho(\rho+2p), \ldots\right),\end{aligned} \tag{10}$$

where $\tilde{T}_\nu^\mu$ can be regarded as the total effective energy–momentum tensor on the brane and $\kappa_n^2 = 8\pi G_n$ denotes Newton's constant. Now we would like to project the total effective energy–momentum tensor $\tilde{T}_\nu^\mu$ to the normal direction of the $(n-1)$-space and have it denoted as T^{ab}. Then, one may define the *work density* by [18,19],

$$W = -\frac{1}{2}T^{ab}h_{ab}, \tag{11}$$

and the *energy-supply* vector is given by,

$$\Psi_a \equiv T_a^b\partial_b\tilde{r} + W\partial_a\tilde{r}. \tag{12}$$

As noted in Ref. [19], the work density at the apparent horizon may be viewed as the work done by the change of the apparent horizon and the energy-supply at the horizon is total energy flow through the apparent horizon. Thus the total change of energy on the apparent horizon can be written as,

$$\nabla E = A\Psi + W\nabla V, \tag{13}$$

where $A = (n-1)\Omega_{n-1}\tilde{r}^{n-2}$ and $V = \Omega_{n-1}\tilde{r}^{n-1}$ are the area and the volume of the $(n-1)$-brane with radius $\tilde{r}$, and $\Omega_{n-1} = \pi^{(n-1)/2}/\Gamma((n-1)/2+1)$ is the volume of an $(n-1)$-dimensional unit ball. Eq. (13) is interpreted as *unified first law* [19].

Let us turn to calculating the heat flow δQ through the apparent horizon during an infinitesimal time interval dt, while keep the volume of the brane stable, namely $\nabla V = 0$. In this sense,

$$dE \equiv A\Psi. \tag{14}$$