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Hawking radiation of linear dilaton black holes

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Abstract

We compute exactly the semi-classical radiation spectrum for a class of non-asymptotically flat charged dilaton black holes, the so-called linear dilaton black holes. In the high frequency regime, the temperature for these black holes generically agrees with the surface gravity result. In the special case where the black hole is massless, we show that, although the surface gravity remains finite, there is no radiation, in agreement with the fact that massless objects cannot radiate.

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Quantum field theory in curved spacetime predicts new phenomena such as particle emission by a black hole [1]. This is due to the fact that the vacuum for a quantum field near the horizon is different from the observer's vacuum at spatial infinity. A distant observer thus receives from a black hole a steady flux of particles exhibiting, in the high frequency regime, a black body spectrum with a temperature proportional to the surface gravity [2]. Although Hawking's original derivation of this black hole evaporation dealt with realistic collapsing black holes, Unruh [3] showed that the same results are obtained when the collapse is replaced by appropriate boundary conditions on the horizon of an eternal black hole. In the semi-classical approximation, the black hole radiation spectrum may be evaluated by computing the Bogoliubov coefficients relating the two vacua. An equivalent procedure is to compute the reflection and absorption coefficients of a wave by the black hole. Usually, the wave equation cannot be solved exactly, and one must resort to match solutions in an overlap region between the near-horizon and asymptotic regions [4,5]. In the special case of the $(2+1)$-dimensional BTZ black hole [6], an exact solution of the wave equation is available, which allows for an exact computation of the radiation spectrum, leading to the Hawking temperature [7–9].

In this Letter, we discuss another case of black holes also allowing for an exact semi-classical computation of their radiation spectrum, that of linear dilaton black hole solutions to Einstein–Maxwell dilaton (EMD) theory in four dimensions. Linear dilaton black holes are a special case of the more general class of non-asymptotically flat black hole solutions to EMD [10,11], which we first briefly present. We discuss the evaporation of these non-asymptotically flat black holes and show that they either collapse to a naked singularity in a finite time, or evaporate in an infinite time. We then specialize to linear dilaton black holes, and outline the analytical computation of their radiation spectrum. For massive black holes, this computation leads, in the high frequency regime, to the same temperature which is obtained from the surface gravity. However in the case of massless extreme black holes, we find that, although the surface gravity remains finite, there is no radiation, in agreement with the fact that a massless object cannot radiate.

EMD is defined by the following action

$$S=\frac{1}{16\pi}\int dx^4\sqrt{-g}\big[R-2\partial_\mu\phi\partial^\mu\phi-e^{-2\alpha\phi}F_{\mu\nu}F^{\mu\nu}\big], \qquad (1)$$

where $F_{\mu\nu}$ is the electromagnetic field, and ϕ is the dilatonic field, with coupling constant α. This theory admits static spher-

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ically symmetric solutions representing black holes. Among these black hole solutions there are asymptotically flat ones [12,13] as well as non-asymptotically flat configurations [10,11]. In the present work, we are interested in the non-asymptotically flat black hole solutions

$$ds^2 = \frac{r^\gamma (r-b)}{r_0^{\gamma+1}} dt^2 - \frac{r_0^{\gamma+1}}{r^\gamma (r-b)} \{dr^2 + r(r-b)\, d\Omega^2\}, \quad (2)$$

$$F = \sqrt{\frac{1+\gamma}{2}} \frac{\nu}{r_0} dr \wedge dt, \qquad e^{2\alpha\phi} = \nu^2 \left(\frac{r}{r_0}\right)^{1-\gamma} \quad (3)$$

with

$$\gamma = \frac{1-\alpha^2}{1+\alpha^2}. \quad (4)$$

The constants b and r_0 are related to the mass and to the electric charge of the black hole through

$$M = (1-\gamma)b/4, \qquad Q = \sqrt{\frac{1+\gamma}{2}} \frac{r_0}{\nu}. \quad (5)$$

The solutions (2), (3) interpolate between the Schwarzschild solution for $\gamma = -1$ ($\alpha^2 \to \infty$) and the Bertotti–Robinson solution for $\gamma = +1$ ($\alpha^2 = 0$). For $\gamma = 0$ ($\alpha^2 = 1$) the dilaton ϕ is asymptotically linear in the 'tortoise' coordinate $\ln(r-b)$, hence the term "linear dilaton black hole" for the corresponding solution [14]. For all $\gamma < 1$, the horizon at $r = b$ hides the singularity at $r = 0$ if $b > 0$, while in the extreme black hole case $b = 0$ the horizon coincides with the singularity. This is a curious case, with vanishing mass but a finite electric charge. For $-1 < \gamma < 0$ ($\alpha^2 > 1$) the central singularity is timelike and clearly naked [11]. On the other hand, for $0 \leqslant \gamma < 1$ ($0 < \alpha^2 \leqslant 1$), the central singularity is null and marginally trapped [15], so that signals coming from the centre never reach external observers. Thus in this case, extreme black holes can be still considered as black holes indeed.

The statistical Hawking temperature of the black holes (2), computed as usual by dividing the surface gravity by 2π is given by

$$T_H = \frac{1}{4\pi} \frac{b^\gamma}{r_0^{1+\gamma}}. \quad (6)$$

It is finite for all γ if $b \neq 0$. For $b = 0$ and $-1 < \gamma < 0$ (naked singularity), the temperature is infinite, while for $b = 0$ and $0 < \gamma < 1$ (extreme black hole), the temperature vanishes.

The case $b = \gamma = 0$ is intriguing. Although this an extreme black hole, the situation is different from that of asymptotically flat extreme black holes. The near-horizon Euclidean extreme Reissner–Nordström geometry is cylindrical, rather than conical, so that its statistical temperature is arbitrary, contrary to the zero value derived from surface gravity [16]. In the present case the two-dimensional Euclidean continuation of the metric (2) with $\gamma = 0$ clearly has a conical singularity at $r = b$ for all values of b, including $b = 0$, leading for this particular extreme black hole to the finite temperature $T_H = 1/4\pi r_0$, in agreement with the value (6). However this result is questionable. A black hole with pointlike horizon and zero mass clearly cannot radiate, so one should rather expect its temperature to be zero. We will return to this question presently.

As black holes (2) radiate, they loose mass according to Stefan's law

$$\frac{dM}{dt} = -\sigma A_h T_H^4, \quad (7)$$

where σ is Stefan's constant, and $A_h = 4\pi r_0^{1+\gamma} b^{1-\gamma}$ is the horizon area. Assuming that only electrically neutral quanta are radiated, (7) implies that the horizon area decreases according to

$$\frac{db}{dt} = -\frac{4\sigma}{(4\pi)^3(1-\gamma)} r_0^{-3(1+\gamma)} b^{1+3\gamma}, \quad (8)$$

which is solved by

$$b(t) = r_0 \left(\frac{\gamma c}{1-\gamma} \frac{t-t_0}{r_0^3}\right)^{-1/3\gamma} \qquad (\gamma \neq 0),$$
$$b(t) = r_0 \exp\left(-\frac{c}{3} \frac{t-t_0}{r_0^3}\right) \quad (\gamma = 0), \quad (9)$$

where $c = 3\sigma/16\pi^3$, and t_0 is an integration constant. The outcome depends on the sign of γ. For $\gamma < 0$, the Hawking temperature increases with decreasing mass and the black hole collapses to a naked singularity (or evaporates away altogether in the Schwarzschild case $\gamma = -1$) in a finite time according to $b \sim (t_0 - t)^{1/3|\gamma|}$. On the other hand, for $\gamma \geqslant 0$, the Hawking temperature decreases (or is constant for $\gamma = 0$) with decreasing mass, and the black hole evaporates in an infinite time, reaching the extreme black hole state $b = 0$ only asymptotically.

We now proceed to a more precise evaluation of the temperature of non-asymptotically flat black holes from the study of wave scattering in these spacetimes. The wave equation

$$\nabla^2 \phi = 0 \quad (10)$$

does not generically allow for an exact solution in the spacetimes (2). However, it can be solved analytically [14] in the case of linear dilaton black holes with $\gamma = 0$ and $b \neq 0$, with the metric

$$ds^2 = \frac{r-b}{r_0} dt^2 - \frac{r_0}{r-b} \{dr^2 + r(r-b)\, d\Omega^2\}. \quad (11)$$

Considering the harmonic eigenmodes

$$\phi(x) = \psi(r,t) Y_{lm}(\theta, \varphi), \quad \psi(r,t) = R(r) e^{-i\omega t}, \quad (12)$$

we obtain the following radial equation:

$$\partial_r \big(r(r-b)\partial_r R\big) + \left(\bar{\omega}^2 \frac{r}{r-b} - l(l+1)\right) R = 0 \quad (13)$$

($\bar{\omega}^2 \equiv \omega^2 r_0^2$). Putting

$$y = \frac{b-r}{b}, \qquad R = y^{i\bar{\omega}} f, \quad (14)$$

reduces (13) to the equation

$$y(1-y)\partial_y^2 f + \big(1 + 2i\bar{\omega} - 2(1+i\bar{\omega})y\big)\partial_y f$$
$$+ \big(\bar{\omega}^2 - i\bar{\omega} - \bar{\lambda}^2 - 1/4\big) f = 0, \quad (15)$$