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Generalized Chaplygin gas model: Constraints from Hubble parameter versus redshift data

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Abstract

We examine observational constraints on the generalized Chaplygin gas (GCG) model for dark energy from the 9 Hubble parameter data points, the 115 SNLS Sne Ia data and the size of baryonic acoustic oscillation peak at redshift, $z = 0.35$. At a 95.4% confidence level, a combination of three data sets gives $0.67 \leqslant A_s \leqslant 0.83$ and $-0.21 \leqslant \alpha \leqslant 0.42$, which is within the allowed parameters ranges of the GCG as a candidate of the unified dark matter and dark energy. It is found that the standard Chaplygin gas model ($\alpha = 1$) is ruled out by these data at the 99.7% confidence level.

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1. Introduction

Many astrophysical and cosmological observations, including Type Ia Supernovae (Sne Ia) [1] and cosmic microwave background radiation (CMBR) [2,3] etc., indicated that the universe is undergoing an accelerating expansion. Many works have being done in order to explain this discovery. Some people attribute the observed acceleration to a possible breakdown of our understanding of the laws of gravitation, thus they attempted to modify the Friedmann equation [4,5]. However, many more think that the cosmic acceleration is driven by an exotic energy component with the negative pressure in the universe, named dark energy, which at late times dominates the total energy density of our universe and accelerates the cosmic expansion. Up to now there are many candidates of dark energy, such as the cosmological constant Λ [6], quintessence [7], phantom [8] and quintom [9] etc.

Recently an interesting model of dark energy, named the Chaplygin gas, was proposed by Kamenshchik et al. [10]. This model is characterized by an exotic equation of state

$$p_{\rm ch} = -\frac{A}{\rho_{\rm ch}^{\alpha}} \tag{1}$$

with a positive constant A and $\alpha = 1$. Progress has been made toward generalizing these model parameters. In this regard, Bento et al. generalized parameter α from 1 to an arbitrary constant in Ref. [11], and this generalized model was called the generalized Chaplygin gas (GCG) model and can be obtained from a generalized version of the Born–Infeld action. For $\alpha = 0$ the GCG model behaves like the scenario with cold dark matter plus a cosmological constant.

Inserting the above equation of state of the GCG into the energy conservation equation, it is easy to obtain

$$\rho_{\rm ch} = \rho_{\rm ch0}\left(A_s + \frac{1-A_s}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}, \tag{2}$$

where $\rho_{\rm ch0}$ is the present energy density of the GCG and $A_s \equiv A/\rho_{\rm ch0}^{1+\alpha}$. It is worth noting that, when $0 < A_s < 1$, the GCG model smoothly interpolates between a non-relativistic matter phase ($\rho_{\rm ch} \propto a^{-3}$) in the past and at late times a negative pressure dark energy regime ($\rho_{\rm ch} = -p_{\rm ch}$). As a result of

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this interesting feature, the GCG model has been proposed as a model of the unified dark matter and dark energy (UDME). Meanwhile, for $A_s = 0$ the GCG behaves always like matter while for $A_s = 1$ it behaves always like a cosmological constant.

The GCG model, thus, has been the subject of great interest and many authors have attempted to constrain this UDME model by using various observational data, such as the Sne Ia [12–17], the CMBR [17–19], the gamma-ray bursts [20], the gravitational lensing [14,16,21], the X-ray gas mass fraction of clusters [13–15], the large scale structure [17,22], and the age of high-redshift objects [23].

In this Letter we shall consider the new observational constraints on the parameter space of the GCG for a flat universe by using a measurement of the Hubble parameter as a function of redshift [24], the new 115 Sne Ia data released by the Supernova Legacy Survey (SNLS) Collaboration recently [26] and the baryonic acoustic oscillation (BAO) peak detected in the large-scale correlation function of luminous red galaxies from Sloan Digital Sky Survey (SDSS) [27]. We perform a combined analysis of three databases and find that the degeneracy between A_s and α is broken. At a 95.4% confidence level we obtain a strong constraint on the GCG model parameters: $0.67 \leqslant A_s \leqslant 0.83$ and $-0.21 \leqslant \alpha \leqslant 0.42$, a parameter range within which the GCG model could be taken as a candidate of UDME and the pure Chaplygin gas model could be ruled out.

2. Constraint from the Hubble parameter as a function of redshift

Last year, based on differential ages of passively evolving galaxies determined from the Gemini Deep Deep Survey [28] and archival data [29], Simon et al. [30] gave an estimate for the Hubble parameter as a function of the redshift z,

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}, \tag{3}$$

where t is the time. They obtained 9 data points of $H(z)$ at redshift z_i and used the estimated $H(z)$ to constrain the dark energy potential. Later these 9 data points were used to constrain parameters of holographic dark energy model [31] and parameters of the ΛCDM, XCDM and ϕCDM models [32]. Here we will use this data to constrain the GCG model.

For a flat universe containing only the baryonic matter and the GCG, the Friedmann equation can be expressed as

$$H^2(H_0, A_s, \alpha, z) = H_0^2 E^2(A_s, \alpha, z), \tag{4}$$

where

$$E(A_s, \alpha, z) = \big[\Omega_b(1+z)^3 + (1-\Omega_b) \times \big(A_s + (1-A_s)(1+z)^{3(1+\alpha)}\big)^{\frac{1}{1+\alpha}}\big]^{1/2}, \tag{5}$$

Ω_b is the present dimensionless density parameter of baryonic matter and $H_0 = 100h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ is present Hubble constant. The Hubble Space Telescope key projects give $h = 0.72 \pm 0.08$ [33] and the WMAP observations give $\Omega_b h^2 = 0.0233 \pm 0.0008$ [3]. The best fit values for model parameters A_s, α and constant H_0 can be determined by minimizing

$$\chi^2(H_0, A_s, \alpha) = \sum_{i=1}^{9} \frac{[H(H_0, A_s, \alpha, z_i) - H_{\mathrm{obs}}(z_i)]^2}{\sigma^2(z_i)}. \tag{6}$$

Since we are interested in the model parameters, H_0 becomes a nuisance parameter. We marginalize over H_0 to get the probability distribution function of A_s and α: $L(A_s, \alpha) = \int dH_0\, P(H_0) e^{-\chi^2(H_0, A_s, \alpha)/2}$, where $P(H_0)$ is the prior distribution function for the present Hubble constant. In this Letter a Gaussian priors $H_0 = 72 \pm 8\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ is considered.

In Fig. 1, we show the data of the Hubble parameter plotted as a function of redshift for the case $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. Fig. 2 shows the results of our statistical analysis for the Hubble parameter data. Confidence contours (68.3, 95.4 and 99.7%) in the A_s–α plane are displayed by considering the Hubble parameter measurements discussed above. The best fit happens at $A_s = 0.82$ and $\alpha = 0.71$. It is very clear that two model parameters, A_s and α, are degenerate.

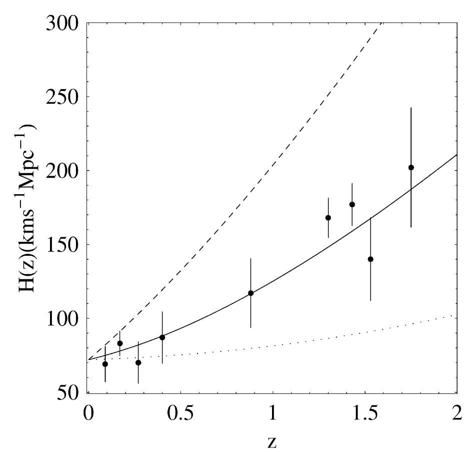

Fig. 1. The Hubble parameters $H(z)$ as a function of z for the case $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. The solid curve corresponds to our best fit to 9 Hubble parameter data plus SNLS SNe Ia data and SDSS baryonic acoustic oscillation peak with $A_s = 0.75$, $\alpha = 0.05$. The dotted line and dashed line correspond to $A_s = 1.0$ and $A_s = 0.0$, respectively.

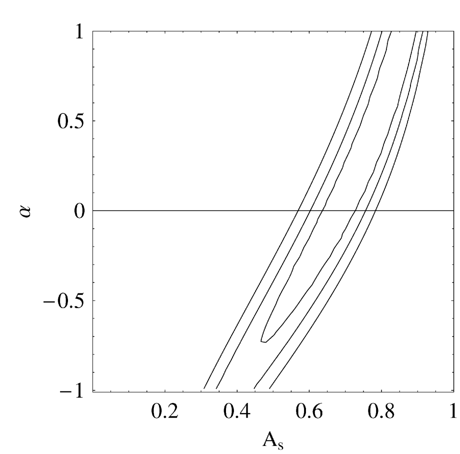

Fig. 2. The 68.3, 95.4 and 99.7% confidence level contours for A_s versus α from the measurement of Hubble parameter with a Gaussian priors $H_0 = 72 \pm 8\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. The best fit happens at $A_s = 0.82$ and $\alpha = 0.71$.