Physics Letters B 677 (2009) 272–277

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Infrared safety in factorized hard scattering cross-sections

Andrew Hornig, Christopher Lee *, Grigory Ovanesyan

Ernest Orlando Lawrence Berkeley National Laboratory and University of California, Berkeley, CA 94720, USA

ARTICLE INFO

Article history:
Received 27 January 2009
Received in revised form 13 May 2009
Accepted 19 May 2009
Available online 23 May 2009
Editor: B. Grinstein

PACS:
12.38.Bx
12.39.St
13.66.Bc
13.87.-a

Keywords:
Factorization
Soft collinear effective theory
Jets
Event shapes

ABSTRACT

The rules of soft-collinear effective theory can be used naïvely to write hard scattering cross-sections as convolutions of separate hard, jet, and soft functions. One condition required to guarantee the validity of such a factorization is the infrared safety of these functions in perturbation theory. Using e^+e^- angularity distributions as an example, we propose and illustrate an intuitive method to test this infrared safety at one loop. We look for regions of integration in the sum of Feynman diagrams contributing to the jet and soft functions where the integrals become infrared divergent. Our analysis is independent of an explicit infrared regulator, clarifies how to distinguish infrared and ultraviolet singularities in pure dimensional regularization, and demonstrates the necessity of taking zero-bins into account to obtain infrared-safe jet functions.

1. Introduction

Factorization restores predictive power to calculations in Quantum Chromodynamics (QCD) which cannot be carried out exactly due to the contributions of nonperturbative effects. By separating perturbatively-calculable and nonperturbative contributions to observables in QCD and relating the nonperturbative contributions to different observables to each other, we gain the ability to make real predictions.

Proving factorization rigorously is a technically challenging undertaking, which traditionally has been formulated in full QCD [1,2]. More recently, many formal elements of these factorization proofs, such as power counting, gauge invariance, the organization of soft gluons into eikonal Wilson lines, and their decoupling from collinear modes, have been organized in the framework of soft-collinear effective theory (SCET) [3–6]. These generic properties of the effective theory allow one to write at least nominally a formula "factorized" into collinear (jet) and soft functions for an arbitrary hard scattering cross-section in which strongly-interacting light-like particles participate [7]. Examples are the factorization of a large class of two-jet event shape distributions in e^+e^- annihilations to light quark jets [8–10], jet mass distributions for e^+e^- to top quark jets [11], or arbitrary jet cross-sections in pp collisions independently of the choice of actual jet algorithm or observable [12]. While the formalism of SCET leads directly to expressing these observables as convolutions of separate hard, jet, and soft functions, blind use of this procedure without considering further specific properties of each chosen observable can hide whether factorization truly holds in a particular case or not.

An ideal set of observables for which to examine factorizability is the set of angularities τ_a [13], which are two-jet e^+e^- event shapes dependent on a tunable parameter a controlling how sensitive the event shape is to radiation along the jet axes or at wider angles. Varying a between 0 and 1 interpolates between the thrust [14,15] and jet broadening [16] event shapes, but a can take any value $-\infty < a < 2$ and give an infrared-safe observable in QCD. Angularities are known to be factorizable, however, only for $a < 1$ [13]. For events $e^+e^- \to X$, the angularity of a final state X is

$$\tau_a(X) = \frac{1}{Q}\sum_{i\in X} E_i \sin^a\theta_i (1-\cos\theta_i)^{1-a}$$
$$= \frac{1}{Q}\sum_{i\in X} |\mathbf{p}_i^\perp| e^{-|\eta_i|(1-a)}, \tag{1}$$

where in the first form E_i is the energy of particle i and θ_i is the angle between its momentum and the thrust axis of X. In the

* Corresponding author.
E-mail addresses: ahornig@berkeley.edu (A. Hornig), clee@berkeley.edu (C. Lee), ovanesyan@berkeley.edu (G. Ovanesyan).

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doi:10.1016/j.physletb.2009.05.039

second form, $\mathbf{p}_i^\perp$ is the momentum of particle i transverse to the thrust axis, and η_i is its rapidity with respect to the direction of the thrust axis. We assume all final-state particles are massless.

In a separate publication, using SCET, we calculate the angularity jet and soft functions to next-to-leading order in the strong coupling α_s, resum large logarithms using renormalization group evolution, and model the nonperturbative soft function in a way that avoids renormalon ambiguities [17].

In this Letter, using angularity distributions as an example, we describe a simple, intuitive method for testing the validity of a factorization theorem deduced from the simple rules of SCET. We begin by naïvely presuming the factorizability of a given observable and then attempt to calculate perturbatively the one-loop jet and soft functions. If the factorization holds, each of these functions should be infrared-safe. If they are not, we learn immediately that the factorization breaks down.

Perturbative infrared-safety of jet and soft functions is not, of course, by itself sufficient to guarantee validity of the proposed factorization theorem. The size of power corrections must also be taken into account. The methods we describe in this Letter address only the former issue, not the latter. (Power corrections for angularity distributions and their implications for factorizability were studied in [10,13,18].) However, our method is a quick and direct way to narrow down the class of observables for which a "generic" factorization deduced from SCET (e.g. [12]) could actually be valid.

Our analysis also sheds light on some issues related to infrared divergences in effective theory loop integrals. Finding a tractable regulator in SCET that suitably controls all infrared divergences has been very challenging (see, e.g., [19,20]). Care is also required to define the effective theory such that it avoids double-counting momentum regions and infrared divergences of full theory diagrams. The procedure of zero-bin subtraction has been proposed to eliminate such double-counting [21].

We will address each of these issues without explicit calculation of the jet and soft loop integrals or use of an explicit infrared regulator. Instead we just examine the regions of integration surviving in the sum over all relevant diagrams. We will work in pure dimensional regularization, and learn how to identify $1/\epsilon$ poles as infrared or ultraviolet in origin, clarifying the contribution made by scaleless integrals which are formally zero. We will thus conclude that the analysis is independent of the choice of any explicit IR regulator. In the process, we demonstrate the crucial role of zero-bin subtractions in obtaining physically-consistent, infrared-safe jet functions in angularity distributions for all $a < 1$. The ideas and methods illustrated through our discussion of angularity distributions are more generally applicable to other observables as well.

2. Angularity distributions in SCET

The factorization theorem for the angularity distributions $d\sigma/d\tau_a$ takes the form,

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a} = H(Q;\mu)\int d\tau_a^n\, d\tau_a^{\bar n}\, d\tau_a^s\,\delta\big(\tau_a-\tau_a^n-\tau_a^{\bar n}-\tau_a^s\big) \times J_a^n\big(\tau_a^n;\mu\big)J_a^{\bar n}\big(\tau_a^{\bar n};\mu\big)S_a\big(\tau_a^s;\mu\big), \tag{2}$$

where σ_0 is the total $e^+e^-\to q\bar q$ Born cross-section, H is a hard function given in the effective theory by the square of a matching coefficient dependent only on short-distance effects, $J_a^{n,\bar n}$ are jet functions dependent on the partonic branching and evolution of each of the two back-to-back final-state jets, and S_a is a soft function dependent on the low energy radiation from the jets and the color exchange between them. All the functions depend on the factorization scale μ, with this dependence cancelling in the full cross-section. The factorization theorem Eq. (2) for angularity distributions has been proved in full QCD [13] and in SCET [10, 18], for $a<1$, where this condition was derived from the size of power corrections induced by replacing the thrust axis implicit in Eq. (1) with the collinear jet axis $\mathbf{n}$ [10,13]. Our attempt to calculate perturbatively the jet and soft functions in Eq. (2) will provide a complementary way to deduce this condition and an intuitive explanation of the absence of infrared divergences in the jet and soft functions for $a<1$ and their appearance for $a\geqslant 1$.

Collinear and soft modes in SCET are distinguished by the scaling of the momenta of the particles they describe. The light-cone components $p=(n\cdot p,\bar n\cdot p,p_\perp)$ of collinear modes, where $n,\bar n$ are light-cone vectors in the $\pm z$ directions, scale as $Q(\lambda^2,1,\lambda)$ or $Q(1,\lambda^2,\lambda)$, and soft modes as $Q(\lambda^2,\lambda^2,\lambda^2)$. Q is the hard energy scale in the process being considered (here, the center-of-mass energy in e^+e^- collisions), and λ is a small ratio of energy scales, here $\lambda=\sqrt{\Lambda_{\rm QCD}/Q}$. Collinear momenta p_c are split into a "label" piece $\tilde p_c$ containing the order Q and $Q\lambda$ momenta, and a "residual" piece k_c all of whose components are order $Q\lambda^2$. A redefinition of the collinear fields through multiplication by soft Wilson lines decouples soft and collinear modes in the SCET Lagrangian to leading order in λ [6].

The soft function S_a in Eq. (2) is defined by

$$S_a\big(\tau_a^s;\mu\big)=\frac{1}{N_C}\,{\rm Tr}\langle 0|\bar Y_{\bar n}^\dagger(0)Y_n^\dagger(0)\delta\big(\tau_a^s-\hat\tau_a^s\big)Y_n(0)\bar Y_{\bar n}(0)|0\rangle, \tag{3}$$

and the jet functions $J_a^{n,\bar n}$ by

$$J_a^n\big(\tau_a^n,\mu\big)\left(\frac{\not n}{2}\right)_{\alpha\beta}=\frac{1}{N_C}\,{\rm Tr}\int\frac{dl^+}{2\pi}\int d^4x\,e^{il\cdot x} \times\langle 0|\chi_{n,Q}(x)_\alpha\delta\big(\tau_a^n-\hat\tau_a^n\big)\bar\chi_{n,Q}(0)_\beta|0\rangle, \tag{4a}$$

$$J_a^{\bar n}\big(\tau_a^{\bar n},\mu\big)\left(\frac{\not{\bar n}}{2}\right)_{\alpha\beta}=\frac{1}{N_C}\,{\rm Tr}\int\frac{dk^-}{2\pi}\int d^4x\,e^{ik\cdot x} \times\langle 0|\bar\chi_{\bar n,-Q}(x)_\beta\delta\big(\tau_a^{\bar n}-\hat\tau_a^{\bar n}\big)\chi_{\bar n,-Q}(0)_\alpha|0\rangle. \tag{4b}$$

The traces are over colors, the light-cone momenta are defined $l^+=n\cdot l$ and $k^-=\bar n\cdot k$, and the subscripts $\pm Q$ on the jet fields in Eq. (4) specify that they create jets with total label momenta $Qn/2$ and $Q\bar n/2$ [5]. The soft Wilson line Y_n in the soft function is the path-ordered exponential of soft gluons,

$$Y_n(z)=P\exp\left[ig\int_0^\infty ds\,n\cdot A_s(ns+z)\right], \tag{5}$$

and similarly for $\bar Y_{\bar n}$, with the bar denoting the anti-fundamental representation. The fields $\chi_{n,\bar n}$ in the jet function are the product of collinear Wilson lines and quarks, $\chi_n=W_n^\dagger\xi_n$, where

$$W_n(z)=P\exp\left[ig\int_0^\infty ds\,\bar n\cdot A_n(\bar ns+z)\right], \tag{6}$$

and similarly for $W_{\bar n}$. The operator $\hat\tau_a$ acts on final states $|X\rangle$ according to

$$\hat\tau_a|X\rangle=\frac{1}{Q}\sum_{i\in X}\big|\mathbf{p}_i^\perp\big|e^{-|\eta_i|(1-a)}|X\rangle, \tag{7}$$

and is constructed from the energy–momentum tensor $T_{\mu\nu}$ [22], and the operators $\hat\tau_a^{n,\bar n,s}$ in Eqs. (3) and (4) are constructed by