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Taub–NUT black holes in third order Lovelock gravity

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ABSTRACT

We consider the existence of Taub–NUT solutions in third order Lovelock gravity with cosmological constant, and obtain the general form of these solutions in eight dimensions. We find that, as in the case of Gauss–Bonnet gravity and in contrast with the Taub–NUT solutions of Einstein gravity, the metric function depends on the specific form of the base factors on which one constructs the circle fibration. Thus, one may say that the independence of the NUT solutions on the geometry of the base space is not a robust feature of all generally covariant theories of gravity and is peculiar to Einstein gravity. We find that when Einstein gravity admits non-extremal NUT solutions with no curvature singularity at $r = N$, then there exists a non-extremal NUT solution in third order Lovelock gravity. In 8-dimensional spacetime, this happens when the metric of the base space is chosen to be $\mathbb{CP}^3$. Indeed, third order Lovelock gravity does not admit non-extreme NUT solutions with any other base space. This is another property which is peculiar to Einstein gravity. We also find that the third order Lovelock gravity admits extremal NUT solution when the base space is $T^2 \times T^2 \times T^2$ or $S^2 \times T^2 \times T^2$. We have extended these observations to two conjectures about the existence of NUT solutions in Lovelock gravity in any even-dimensional spacetime.

1. Introduction

The question as to why the Planck and electroweak scales differ by so many orders of magnitude remains mysterious. In recent years, attempts have been made to address this hierarchy issue within the context of theories with extra spatial dimensions. In higher-dimensional spacetimes even with the assumption of Einstein—that the left-hand side of the field equations is the most general symmetric conserved tensor containing no more than two derivatives of the metric—the field equations need to be generalized. This generalization has been done by Lovelock [1], and he found a second rank symmetric conserved tensor in d dimensions which contains upto second order derivative of the metric. Other higher curvature gravities which have higher derivative terms of the metric, e.g., terms with quartic derivatives, have serious problems with the presence of tachyons and ghosts as well as with perturbative unitarity, while the Lovelock gravity is free of these problems [2].

Many authors have considered the possibility of higher curvature terms in the field equations and how their existence would modify the predictions about the gravitating system. Here, we are interested in the properties of the black holes, and we want to know which properties of the black holes are peculiar to Einstein gravity, and which are robust features of all generally covariant theories of gravity. This fact provide a strong motivation for considering new exact solutions of Lovelock gravity. We show that some properties of NUT solutions are peculiar to Einstein gravity and not robust feature of all generally covariant theories of gravity. Although the nonlinearity of the field equations causes to have a few exact black hole solutions in Lovelock gravity, there are many papers on this subject [3–5]. In this Letter we introduce Taub–NUT metrics in third order Lovelock gravity, and investigate which properties of these kinds of solutions will be modified by considering higher curvature terms in the field equations.

The original four-dimensional solution [6] is only locally asymptotic flat. The spacetime has as a boundary at infinity a twisted S^1 bundle over S^2, instead of simply being $S^1 \times S^2$. There are known extensions of the Taub–NUT solutions to the case when a cosmological constant is present. In this case the asymptotic structure is only locally de Sitter (for positive cosmological constant) or anti-de Sitter (for negative cosmological constant) and the solutions are referred to as Taub–NUT–(A)dS metrics. In general, the Killing vector that

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corresponds to the coordinate that parameterizes the fibre S^1 can have a zero-dimensional fixed point set (called a NUT solution) or a two-dimensional fixed point set (referred to as a 'bolt' solution). Generalizations to higher dimensions follow closely the four-dimensional case [7–9]. Also, these kinds of solutions have been generalized in the presence of electromagnetic field and their thermodynamics have been investigated [10,11]. It is therefore natural to suppose that the generalization of these solutions to the case of Lovelock gravity, which is the low energy limit of supergravity, might provide us with a window on some interesting new corners of M-theory moduli space.

The outline of this Letter is as follows. We give a brief review of the field equations of third order Lovelock gravity in Section 2. In Section 3, we obtain Taub–NUT solutions of third order Lovelock gravity in eight dimensions and then we check the conjectures given in Ref. [4]. We finish this letter with some concluding remarks.

2. Field equations

The vacuum gravitational field equations of third order Lovelock gravity may be written as:

$$\alpha_0 g_{\mu\nu} + \alpha_1 G^{(1)}_{\mu\nu} + \alpha_2 G^{(2)}_{\mu\nu} + \alpha_3 G^{(3)}_{\mu\nu} = 0, \tag{1}$$

where α_i's are Lovelock coefficients, $G^{(1)}_{\mu\nu}$ is just the Einstein tensor, and $G^{(2)}_{\mu\nu}$ and $G^{(3)}_{\mu\nu}$ are the second and third order Lovelock tensors given as

$$G^{(2)}_{\mu\nu} = 2\big(R_{\mu\sigma\kappa\tau}R_{\nu}{}^{\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^{\sigma}{}_{\nu} + RR_{\mu\nu}\big) - \frac{1}{2}\mathcal{L}_2 g_{\mu\nu}, \tag{2}$$

$$\begin{aligned} G^{(3)}_{\mu\nu} = {} & -3\big(4R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\lambda\rho}R^{\lambda}{}_{\nu\tau\mu} - 8R^{\tau\rho}{}_{\lambda\sigma}R^{\sigma\kappa}{}_{\tau\mu}R^{\lambda}{}_{\nu\rho\kappa} + 2R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\lambda\rho}R^{\lambda\rho}{}_{\tau\mu} \\ & - R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\tau\rho}R_{\nu\mu} + 8R^{\tau}{}_{\nu\sigma\rho}R^{\sigma\kappa}{}_{\tau\mu}R^{\rho}{}_{\kappa} + 8R^{\sigma}{}_{\nu\tau\kappa}R^{\tau\rho}{}_{\sigma\mu}R^{\kappa}{}_{\rho} \\ & + 4R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\mu\rho}R^{\rho}{}_{\tau} - 4R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\tau\rho}R^{\rho}{}_{\mu} + 4R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\tau\mu}R_{\nu\rho} + 2RR_{\nu}{}^{\kappa\tau\rho}R_{\tau\rho\kappa\mu} \\ & + 8R^{\tau}{}_{\nu\mu\rho}R^{\rho}{}_{\sigma}R^{\sigma}{}_{\tau} - 8R^{\sigma}{}_{\nu\tau\rho}R^{\tau}{}_{\sigma}R^{\rho}_{\mu} - 8R^{\tau\rho}{}_{\sigma\mu}R^{\sigma}{}_{\tau}R_{\nu\rho} - 4RR^{\tau}{}_{\nu\mu\rho}R^{\rho}{}_{\tau} \\ & + 4R^{\tau\rho}R_{\rho\tau}R_{\nu\mu} - 8R^{\tau}{}_{\nu}R_{\tau\rho}R^{\rho}{}_{\mu} + 4RR_{\nu\rho}R^{\rho}{}_{\mu} - R^2R_{\nu\mu}\big) - \frac{1}{2}\mathcal{L}_3 g_{\mu\nu}. \end{aligned} \tag{3}$$

In Eqs. (2) and (3) $\mathcal{L}_2 = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss–Bonnet Lagrangian and

$$\begin{aligned} \mathcal{L}_3 = {} & 2R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\rho\tau}R^{\rho\tau}{}_{\mu\nu} + 8R^{\mu\nu}{}_{\sigma\rho}R^{\sigma\kappa}{}_{\nu\tau}R^{\rho\tau}{}_{\mu\kappa} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\nu\rho}R^{\rho}{}_{\mu} \\ & + 3RR^{\mu\nu\sigma\kappa}R_{\sigma\kappa\mu\nu} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\mu}R_{\kappa\nu} + 16R^{\mu\nu}R_{\nu\sigma}R^{\sigma}{}_{\mu} - 12RR^{\mu\nu}R_{\mu\nu} + R^3 \end{aligned} \tag{4}$$

is the third order Lovelock Lagrangian. Eq. (1) does not contain the derivative of the curvatures, and therefore the derivatives of the metric higher than two do not appear. In order to have the contribution of all the above terms in the field equation, the dimension of the spacetime should be equal or larger than seven. Here, for simplicity, we consider the NUT solutions of the dimensionally continued gravity in eight dimensions. The dimensionally continued gravity in D dimensions is a special class of the Lovelock gravity, in which the Lovelock coefficients are reduced to two by embedding the Lorentz group $SO(D-1,1)$ into a larger AdS group $SO(D-1,2)$ [12]. By choosing suitable unit, the remaining two fundamental constants can be reduced to one fundamental constant l. Thus, the Lovelock coefficients α_i's can be written as

$$\alpha_0 = -\frac{21}{l^2}, \qquad \alpha_1 = 3, \qquad \alpha_2 = \frac{3l^2}{20}, \qquad \alpha_3 = \frac{l^4}{120}.$$

3. Eight-dimensional solutions

In this section we study the eight-dimensional Taub–NUT solutions of third order Lovelock gravity. In constructing these metrics the idea is to regard the Taub–NUT spacetime as a $U(1)$ fibration over a 6-dimensional base space endowed with an Einstein–Kähler metric $d\Omega_B^2$. Then the Euclidean section of the 8-dimensional Taub–NUT spacetime can be written as:

$$ds^2 = F(r)(d\tau + N\mathcal{A})^2 + F^{-1}(r)\,dr^2 + \big(r^2 - N^2\big)\,d\Omega_B^2, \tag{5}$$

where τ is the coordinate on the fibre S^1 and A has a curvature $F = dA$, which is proportional to some covariantly constant 2-form. Here N is the NUT charge and $F(r)$ is a function of r. The solution will describe a 'NUT' if the fixed point set of the $U(1)$ isometry $\partial/\partial\tau$ (i.e. the points where $F(r) = 0$) is less than 6-dimensional and a 'bolt' if the fixed point set is 6-dimensional. Here, we consider only the cases where all the factor spaces of $\mathcal{B}$ have zero or positive curvature. Thus, the base space $\mathcal{B}$ can be the 6-dimensional space $\mathbb{CP}^3$, a product of three 2-dimensional spaces (T^2 or S^2), or the product of a 4-dimensional space $\mathbb{CP}^2$ with a 2-dimensional one. The 1-forms and metrics of S^2, T^2, $\mathbb{CP}^2$ and $\mathbb{CP}^3$ are [13]

$$\mathcal{A}_{S^2} = 2\cos\theta_i\,d\phi_i, \qquad d\Omega^2_{S^2} = d\theta_i^2 + \sin^2\theta_i\,d\phi_i^2, \tag{6}$$

$$\mathcal{A}_{T^2} = 2\eta_i\,d\zeta_i, \qquad d\Omega^2_{T^2} = d\eta_i^2 + d\zeta_i^2, \tag{7}$$

$$\mathcal{A}_{\mathbb{CP}^2} = 6\sin^2\theta_2\big(d\phi_2 + \sin^2\theta_1\,d\phi_1\big), \tag{8}$$

$$d\Omega^2_{\mathbb{CP}^2} = 6\big\{d\theta_2{}^2 + \sin^2\theta_2\cos^2\theta_2\big(d\phi_2 + \sin^2\theta_1\,d\phi_1\big)^2 + \sin^2\theta_2\big(d\theta_1{}^2 + \sin^2\theta_1\cos^2\theta_1\,d\phi_1^2\big)\big\} \tag{9}$$

$$\mathcal{A}_{\mathbb{CP}^3} = \frac{1}{2}\left(\frac{1}{2}\big(\cos^2\theta_3 - \sin^2\theta_3\big)\,d\phi_3 - \cos^2\theta_3\cos\theta_1 d\phi_1 - \sin^2\theta_3\cos\theta_2\,d\phi_2\right),$$