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Test of the heavy quark–light diquark approximation for baryons with a heavy quark

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ABSTRACT

We check a commonly used approximation in which a baryon with a heavy quark is described as a heavy quark–light diquark system. The heavy quark influences the diquark internal motion reducing the average distance between the two light quarks. Besides, we show how the average distance between the heavy quark and any of the light quarks, and that between the heavy quark and the center of mass of the light diquark, are smaller than the distance between the two light quarks, which seems to contradict the heavy quark–light diquark picture. This latter result is in agreement with expectations from QCD sum rules and lattice QCD calculations. Our results also show that the diquark approximations produces larger masses than the ones obtained in a full calculation.

1. Introduction

Heavy quark symmetry [1–6] (HQS) predicts that in baryons with a heavy quark, and up to corrections in the inverse of the heavy quark mass, the light degrees of freedom quantum numbers are well defined, in particular the total spin of the light degrees of freedom is well defined. This prediction has been taken in different calculations as the basis for treating the light quark subsystem as a diquark, and the baryon as a heavy quark–light diquark (HQLD) system [7–16]. This HQS prediction does not imply though that the orbital motion of the two light quarks is not affected by the presence of the heavy quark as it seems to be implicit in the HQLD approximation.[1] Very recently the diquark structure of heavy baryons have been analyzed in Λ_c production in heavy ion collisions [17] where its enhanced yield is seen as a signal for the existence of light diquark correlations both in the quark gluon plasma and the heavy baryon.

In Ref. [18], using a light-front constituent quark model and a Gaussian ansatz for the wave function, the authors studied the dependence of the Isgur–Wise function [4] on the baryon structure. They found very different behaviors for a diquark-like configuration (the heavy quark is far from the center of mass of the light quarks) or a collinear-type configuration (the heavy quark is close to the center of mass of the light quarks). Comparison of the results with QCD sum rules [19] and lattice QCD calculations [20] suggested a clear dominance of the collinear-type configurations. This result seems to go against the HQLD approximation.

Here we plan to check the validity of the HQLD approximation, that we formulate in next section, by looking at heavy baryons masses and quark distributions inside baryons composed of a heavy quark (b or c) and two light quarks. We shall compare the predictions obtained within that approximation with the ones obtained in a full calculation where the effect of the heavy quark on the light diquark is not neglected. For that purpose we shall use the nonrelativistic quark model and the full wave functions described in Ref. [21]. In that reference we took advantage of HQS constraints on the spin of the light degrees of freedom to solve the full nonrelativistic three-body problem by means of a simple variational ansatz. The scheme of Ref. [21] for the wave functions

Table 1
Summary of the quantum numbers of ground-state heavy baryons containing a single heavy quark. I, and S_l^π are the isospin, and the spin parity of the light degrees of freedom and S, J^P are the strangeness and the spin parity of the baryon. We also give the quark content where l denotes a light quark of flavor u or d

Baryon	S	J^P	I	S_l^π	Quark content	Baryon	S	J^P	I	S_l^π	Quark content
Λ_c	0	$\frac{1}{2}^+$	0	0^+	udc	Λ_b	0	$\frac{1}{2}^+$	0	0^+	udb
Σ_c	0	$\frac{1}{2}^+$	1	1^+	llc	Σ_b	0	$\frac{1}{2}^+$	1	1^+	llb
Σ_c^*	0	$\frac{3}{2}^+$	1	1^+	llc	Σ_b^*	0	$\frac{3}{2}^+$	1	1^+	llb
Ξ_c	−1	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	lsc	Ξ_b	−1	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	lsb
Ξ_c'	−1	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	lsc	Ξ_b'	−1	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	lsb
Ξ_c^*	−1	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	lsc	Ξ_b^*	−1	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	lsb
Ω_c	−2	$\frac{1}{2}^+$	0	1^+	ssc	Ω_b	−2	$\frac{1}{2}^+$	0	1^+	ssb
Ω_c^*	−2	$\frac{3}{2}^+$	0	1^+	ssc	Ω_b^*	−2	$\frac{3}{2}^+$	0	1^+	ssb

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[1] Note however that although in the HQLD approximation the light diquark internal structure is not affected by the heavy quark, this structure is commonly taken into account to build up the heavy quark–light diquark interaction.

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Table 2
Masses in MeV obtained with our full calculation in Ref. [21] and with the HQLD approximation (see text for details). In all cases we use the AL1 interquark potential of Refs. [22,23]. We also show experimental masses (isospin average) and lattice estimates when the former are not known. Experimental masses have been taken from Refs. [25], [26] (†) and [27] (§). Lattice estimates (‡) have been taken from Ref. [28]. Note in Ref. [26] what it is actually measured is the mass difference $M_{\Omega_c^*} - M_{\Omega_c}$

Baryon	Full calcul. [21]	HQLD approx.	Exp.	Baryon	Full calcul. [21]	HQLD approx.	Exp.
Λ_c	2295	2317	2286.48 ± 0.14	Λ_b	5643	5663	5624 ± 9
Σ_c	2469	2521	2453.6 ± 0.5	Σ_b	5851	5897	$5812^{\S} \pm 3$
Σ_c^*	2548	2579	2518 ± 2	Σ_b^*	5882	5919	$5833^{\S} \pm 3$
Ξ_c	2474	2501	2469.5 ± 0.6	Ξ_b	5808	5837	$5760^{\ddagger} \pm 70$
Ξ_c'	2578	2629	2577 ± 4	Ξ_b'	5946	5993	$5900^{\ddagger} \pm 70$
Ξ_c^*	2655	2686	2646 ± 1.4	Ξ_b^*	5975	6015	$5900^{\ddagger} \pm 70$
Ω_c	2681	2727	2697.5 ± 2.6	Ω_b	6033	6081	$5990^{\ddagger} \pm 70$
Ω_c^*	2755	2783	$2768.3^{\dagger} \pm 3.2$	Ω_b^*	6063	6104	$6000^{\ddagger} \pm 70$

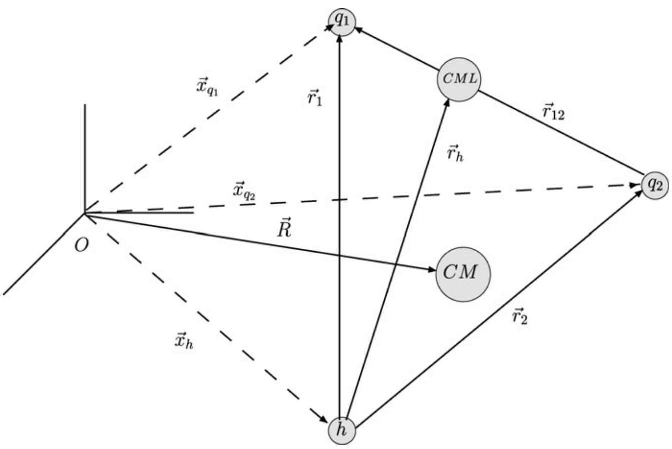

Fig. 1. Definition of different coordinates used through this work. *CM* and *CML* stand for the baryon center of mass and the light quark subsystem center of mass respectively.

reproduced previous results for masses, charge radii..., obtained in Ref. [22] by solving more involved Faddeev equations. The baryons included in that and the present study appear in Table 1. We restrict ourselves to ground-state heavy baryons with total spin $J = 1/2, 3/2$ for which we could assume a zero total orbital angular momentum ($L = 0$).

2. Heavy quark–light diquark approach to a heavy baryon

The set of coordinates more adequate for a heavy quark–light diquark description are the Jacobi coordinates (see Fig. 1)

$$\vec{R} = \frac{m_{q_1}\vec{x}_{q_1} + m_{q_2}\vec{x}_{q_2} + m_h\vec{x}_h}{m_{q_1} + m_{q_2} + m_h},$$

$$\vec{r}_{12} = \vec{x}_{q_1} - \vec{x}_{q_2}, \qquad \vec{r}_h = \frac{m_{q_1}\vec{x}_{q_1} + m_{q_2}\vec{x}_{q_2}}{m_{q_1} + m_{q_2}} - \vec{x}_h, \tag{1}$$

where $\vec{x}_{q_1}$, $\vec{x}_{q_2}$ and $\vec{x}_h$ represent the positions, with respect to a certain reference frame, of the two light quarks and heavy quark respectively, and similarly m_{q_1}, m_{q_2} and m_h are their masses. The Jacobian coordinates are the center of mass position $\vec{R}$, the relative position between the two light quarks $\vec{r}_{12}$, and the relative position between the two light quark center of mass and the heavy quark $\vec{r}_h$.

In terms of these coordinates the three-body Hamiltonian can be written as

$$H = -\frac{\vec{\nabla}^2_{\vec{R}}}{2\bar{M}} + H^{\rm int}, \qquad \bar{M} = m_{q_1} + m_{q_2} + m_h,$$

$$H^{\rm int} = \bar{M} + H_{q_1q_2} + H_{hq_1q_2}, \tag{2}$$

where $-\frac{\vec{\nabla}^2_{\vec{R}}}{2\bar{M}}$ accounts for the total center of mass free motion.

Table 3
Absolute value square of the $\mathcal{P}$ projection coefficient defined in Eq. (8)

	Λ_c	Σ_c	Σ_c^*	Ξ_c	Ξ_c'	Ξ_c^*	Ω_c	Ω_c^*
$\lvert\mathcal{P}\rvert^2$	0.971	0.943	0.957	0.949	0.926	0.932	0.935	0.961
	Λ_b	Σ_b	Σ_b^*	Ξ_b	Ξ_b'	Ξ_b^*	Ω_b	Ω_b^*
$\lvert\mathcal{P}\rvert^2$	0.949	0.946	0.951	0.924	0.921	0.922	0.935	0.946

Besides $\bar{M}$, the different terms in the internal Hamiltonian $H^{\rm int}$ are

$$H_{q_1q_2} = -\frac{\vec{\nabla}^2_{12}}{2\mu_{q_1q_2}} + V_{q_1q_2}(\vec{r}_{12}, {\rm spin}), \qquad \mu_{q_1q_2} = \frac{m_{q_1}m_{q_2}}{m_{q_1} + m_{q_2}},$$

$$\begin{aligned} H_{hq_1q_2} = {} & -\frac{1}{2}\left(\frac{1}{m_{q_1} + m_{q_2}} + \frac{1}{m_h}\right)\vec{\nabla}^2_h \\ & + V_{q_1h}\left(\vec{r}_h + \frac{m_{q_2}}{m_{q_1} + m_{q_2}}\vec{r}_{12}, {\rm spin}\right) \\ & + V_{q_2h}\left(\vec{r}_h - \frac{m_{q_1}}{m_{q_1} + m_{q_2}}\vec{r}_{12}, {\rm spin}\right) \end{aligned} \tag{3}$$

with $\vec{\nabla}_{12} = \partial/\partial_{\vec{r}_{12}}$, $\vec{\nabla}_h = \partial/\partial_{\vec{r}_h}$ and $V_{qq'}$ the interquark potential that depends on relative distances and spins. Defining now

$$\begin{aligned} H^0_{hq_1q_2} = {} & -\frac{1}{2}\left(\frac{1}{m_{q_1} + m_{q_2}} + \frac{1}{m_h}\right)\vec{\nabla}^2_h + V_{q_1h}(\vec{r}_h, {\rm spin}) \\ & + V_{q_2h}(\vec{r}_h, {\rm spin}) \end{aligned} \tag{4}$$

one could write

$$H^{\rm int} = \bar{M} + H_{q_1q_2} + H^0_{hq_1q_2} + \left(H_{hq_1q_2} - H^0_{hq_1q_2}\right). \tag{5}$$

$H_{q_1q_2}$ is the Hamiltonian for the relative motion of the two light quarks while $H^0_{hq_1q_2}$ is the Hamiltonian for the relative motion of the heavy quark with respect to a pointlike light diquark where the two light quarks are located in their center of mass. Both Hamiltonians are coupled through the term $(H_{hq_1q_2} - H^0_{hq_1q_2})$. This latter term cannot be neglected altogether as the light diquark is not pointlike.

Within the HQLD approximation one assumes that the light diquark internal structure is not disturbed by the presence of the heavy quark. This means to neglect the influence of the term $(H_{hq_1q_2} - H^0_{hq_1q_2})$ in the evaluation of the diquark internal wave function, which therefore will be determined by $H_{q_1q_2}$ alone. However, and since the diquark will have a finite size, the effect of $(H_{hq_1q_2} - H^0_{hq_1q_2})$ has to be taken into account to obtain the r_h dependence of the baryon wave function and its mass. Within this approximation, we will take a baryon wave function given by

$$\Psi^{B,{\rm HQLD}}_{hq_1q_2}(r_{12}, r_h) = \Phi_{q_1q_2}(r_{12}) \cdot F_{hq_1q_2}(r_h), \tag{6}$$

where $\Phi_{q_1q_2}(r_{12})$ is the ground-state wave function for the Hamiltonian $H_{q_1q_2}$ and the given spin configuration. We will determine