Available online at www.sciencedirect.com

ScienceDirect

Physics Letters B 660 (2008) 43–48

PHYSICS LETTERS B

www.elsevier.com/locate/physletb

An application of transverse momentum dependent evolution equations in QCD

Federico A. Ceccopieri [a,*], Luca Trentadue [b]

[a] Dipartimento di Fisica, Università di Parma, Viale delle Scienze, Campus Sud, 43100 Parma, Italy
[b] Dipartimento di Fisica, Universitá di Parma, INFN Gruppo Collegato di Parma, Viale delle Scienze, Campus Sud, 43100 Parma, Italy

Received 25 July 2007; received in revised form 27 November 2007; accepted 29 November 2007

Available online 15 December 2007

Editor: A. Ringwald

Abstract

The properties and behaviour of the solutions of the recently obtained k_t-dependent QCD evolution equations are investigated. When used to reproduce transverse momentum spectra of hadrons in Semi-Inclusive DIS, an encouraging agreement with data is found. The present analysis also supports at the phenomenological level the factorization properties of the Semi-Inclusive DIS cross-sections in terms of k_t-dependent distributions. Further improvements and possible developments of the proposed evolution equations are envisaged.

PACS: 12.38.Bx; 12.38.Cy; 13.60.-r; 13.85.Ni

1. Introduction

In a standard perturbative QCD approach to semi-inclusive processes and in particular to Semi-Inclusive Deep Inelastic Scattering (SIDIS), factorization theorems [1] allow to extract soft hadronic wave functions from high energy reactions data. Such non-perturbative process-independent distributions obey QCD renormalization group equations [2]. In presence of a hard scale, set by the virtuality of the exchanged boson in a Deep Inelastic event, standard parton and fragmentation distributions predict, together with the corresponding process-dependent coefficient functions [3,4], the semi-inclusive cross-sections. These distributions, basic ingredients in all nowadays QCD-calculations, are well suited for studying full inclusive processes, such as Deep Inelastic lepton–hadron Scattering or Drell–Yan process in hadronic collisions. In the recent past, however, it has became increasingly clear that less inclusive distributions, either space-like or time-like, are necessary to deal with a variety of semi-inclusive processes. In particular k_t-dependent distributions acquired particular relevance and a great activity has been registered recently in this research field [5]. In the SIDIS case, for instance, final state hadrons are expected to have a sizeable transverse momentum due to the intrinsic motion of partons into hadrons [6], to the radiative process off the struck parton line [4,7] and to the hadronization of partons into hadrons. Unfortunately transverse momentum is usually integrated over, loosing part of the information which is contained in the experimental cross-sections. For these reasons it would be highly desirable to have the evolution equations for k_t-dependent distributions. Such evolution equations were first proposed in the time-like case in Ref. [8] and then recently extended to space-like kinematics in Ref. [11]. In order to have a complete description of the semi-inclusive cross-sections in terms of the transverse momentum, such a generalization was also performed in the target fragmentation region by introducing properly modified [11] fracture functions [12]. The basic idea behind the k_t-dependent evolution equations can be summarized as follows. Let us consider parton emissions off a active, space-like, parton line. In the collinear limit, at each branching, the generated transverse momentum is negligible. In this limit, however, k_t-ordered diagrams can be shown to give leading logarithmic enhancements to the cross-sections. Since

* Corresponding author.
E-mail addresses: federicoalberto.ceccopieri@fis.unipr.it (F.A. Ceccopieri), luca.trentadue@cern.ch (L. Trentadue).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2007.11.095

such contributions can be resummed by the evolution equations [2], at the end of the radiative process, the interacting parton could possibly have an appreciable transverse momentum. In order to take this perturbative effect into account generalized evolution equations are given which therefore depend, in addition to the standard longitudinal momentum fraction, also on transverse degrees of freedom. When solutions to the evolution equations are used to reproduce the semi-inclusive transverse momentum spectrum, the predictions smoothly interpolate from small to large transverse momenta, this being a signature of well known DIS scaling violations in a semi-inclusive process.

The aim of this work is to offer a preliminar phenomenological study of k_t-dependent evolution equations and to compare it with charged hadron production data in DIS current fragmentation region. All the predictions are given by a handful of phenomenological assumptions. Such predictions, however, are not the result of a fit to the data, and thus strengthen our confidence on the general framework as proposed in Ref. [11].

2. Transverse momentum dependent evolution equations

Ordinary QCD evolution equations at leading logarithm accuracy (LLA) resume terms of the type $\alpha_s^n \log^n(Q^2/\mu_F^2)$ originating from quasi-collinear partons emission configurations. Here μ_F^2 represents the factorization scale. Leading contributions are obtained when the virtualities of the partons in the ladder are strongly ordered. At each branching, the emitting parton thus acquires a transverse momentum relative to its initial direction. The radiative transverse momentum can be taken into account through transverse momentum dependent evolution equations. In the time-like case these read [8]:

$$Q^2 \frac{\partial \mathcal{D}_i^h(z_h, Q^2, \boldsymbol{p}_\perp)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^{1} \frac{du}{u} P_{ij}\big(u, \alpha_s(Q^2)\big) \times \int \frac{d^2 \boldsymbol{l}_\perp}{\pi} \delta\big(u(1-u)Q^2 - \boldsymbol{l}_\perp^2\big) \times \mathcal{D}_j^h\left(\frac{z_h}{u}, Q^2, \boldsymbol{p}_\perp - \frac{z_h}{u}\boldsymbol{l}_\perp\right). \tag{1}$$

Fragmentation functions $\mathcal{D}_i^h(z_h, Q^2, \boldsymbol{p}_\perp)$ of Eq. (1) give the probability to find, at a given scale Q^2, a hadron h with longitudinal momentum fraction z_h and transverse momentum $\boldsymbol{p}_\perp$ relative to the parent parton i. $P_{ij}(u)$ are the time-like splitting functions which, as usual, at LL accuracy, can be interpreted as the probabilities to find a parton of type i inside a parton of type j and are expressed as a power series of the strong running coupling, $P_{ij}(u) = \sum_{n=0} \alpha_s^n(Q^2) P_{ij}^{(n)}(u)$. The order n of the expansion of the splitting function matrix $P_{ij}(u)$ actually sets the accuracy of the evolution equations. The radiative transverse momentum square $\boldsymbol{l}_\perp^2$ generated at each branching satisfies the invariant mass constraint $\boldsymbol{l}_\perp^2 = u(1-u)Q^2$. The transverse arguments of $\mathcal{D}_i^h(z_h, Q^2, \boldsymbol{p}_\perp)$ on r.h.s. of Eq. (1) are derived by taking into account the Lorentz boost of transverse momenta from the emitted parton reference frame to the emitting parton one as can be seen on the left panel of Fig. 1.

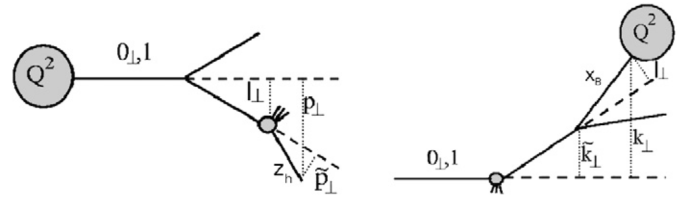

Fig. 1. Boost of transverse momenta. Left panel: a time-like off-shell parton generated in a hard process, the grey blob, emits a daughter parton and acquires a transverse momentum $\boldsymbol{l}_\perp$ relative to its initial direction with $\tilde{\boldsymbol{p}}_\perp = \boldsymbol{p}_\perp - \frac{z_h}{u}\boldsymbol{l}_\perp$. The small blob symbolizes the iteration of such emissions. Right panel: the analogue as before in the space-like case with $\tilde{\boldsymbol{k}}_\perp = (\boldsymbol{k}_\perp - \boldsymbol{l}_\perp)/u$.

The unintegrated distributions fulfill the normalization condition:

$$\int_{\boldsymbol{p}_\perp^2 \leqslant Q^2} d^2\boldsymbol{p}_\perp \mathcal{D}_i^h\big(z_h, Q^2, \boldsymbol{p}_\perp\big) = \mathcal{D}_i^h\big(z_h, Q^2\big). \tag{2}$$

This property guarantees that we can recover ordinary integrated distributions from unintegrated ones. The opposite statement however is not valid since Eq. (1) contains new physical information. In analogy to the time-like case we consider now a initial state parton p in a incoming proton P which undergoes a hard collision, the reference frame being aligned along the incoming proton axis. We thus generalize Eq. (1) to the space-like case [11]:

$$Q^2 \frac{\partial \mathcal{F}_P^i(x_B, Q^2, \boldsymbol{k}_\perp)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^{1} \frac{du}{u^3} P_{ji}\big(u, \alpha_s(Q^2)\big) \times \int \frac{d^2 \boldsymbol{l}_\perp}{\pi} \delta\big((1-u)Q^2 - \boldsymbol{l}_\perp^2\big) \times \mathcal{F}_P^j\left(\frac{x_B}{u}, Q^2, \frac{\boldsymbol{k}_\perp - \boldsymbol{l}_\perp}{u}\right). \tag{3}$$

Parton distribution functions $\mathcal{F}_P^i(x_B, Q^2, \boldsymbol{k}_\perp)$ in Eq. (3) give the probability to find, at a given scale Q^2, a parton i with longitudinal momentum fraction x_B and transverse momentum $\boldsymbol{k}_\perp$ relative to the parent hadron, see the right panel of Fig. 1. The unintegrated distributions fulfill a condition analogous to the one in Eq. (2):

$$\int_{\boldsymbol{k}_\perp^2 \leqslant Q^2} d^2\boldsymbol{k}_\perp \mathcal{F}_P^i\big(x_B, Q^2, \boldsymbol{k}_\perp\big) = \mathcal{F}_P^i\big(x_B, Q^2\big). \tag{4}$$

Transverse momentum dependent parton distributions in Eqs. (2) and (4) were originally introduced in Ref. [9]. An extensive discussion of UV renormalization and gauge dependence of these distributions can be found in Ref. [8]. Furthermore, the integration regions in Eqs. (2) and (4), are dictated by consistency with the leading logarithmic approximation we