Progress in Particle and Nuclear Physics 90 (2016) 75–124

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

# Nonperturbative light-front Hamiltonian methods

J.R. Hiller

*Department of Physics and Astronomy, University of Minnesota-Duluth, Duluth, MN 55812, USA*

ARTICLE INFO

*Article history:*
Available online 23 June 2016

*Keywords:*
Light-front quantization
Nonperturbative Hamiltonian methods
Pauli–Villars regularization
Quantum chromodynamics

ABSTRACT

We examine the current state-of-the-art in nonperturbative calculations done with Hamiltonians constructed in light-front quantization of various field theories. The language of light-front quantization is introduced, and important (numerical) techniques, such as Pauli–Villars regularization, discrete light-cone quantization, basis light-front quantization, the light-front coupled-cluster method, the renormalization group procedure for effective particles, sector-dependent renormalization, and the Lanczos diagonalization method, are surveyed. Specific applications are discussed for quenched scalar Yukawa theory, $\phi^4$ theory, ordinary Yukawa theory, supersymmetric Yang–Mills theory, quantum electrodynamics, and quantum chromodynamics. The content should serve as an introduction to these methods for anyone interested in doing such calculations and as a rallying point for those who wish to solve quantum chromodynamics in terms of wave functions rather than random samplings of Euclidean field configurations.



**Contents**



*E-mail address:* jhiller@d.umn.edu.

http://dx.doi.org/10.1016/j.ppnp.2016.06.002

## 1. Introduction

After many years in gestation, light-front quantization [1–5] is now poised as a viable tool for the nonperturbative solution of quantum chromodynamics (QCD) [6]. This will establish an approach complementary to lattice gauge theory [7], one where wave functions return to their usual central role. Observables can then be computed as expectation values. In addition, the method is formulated in Minkowski space–time, rather than the Euclidean space–time of lattice theory, making time-like quantities more readily accessible. In comparison with equal-time quantization, use of the light-front affords boost-invariant wave functions without spurious vacuum contributions.

The purpose here is not to review the historical development of light-front methods; this is done quite nicely elsewhere [2]. The purpose is instead to summarize the state of the art in nonperturbative light-front calculations, in particular those aspects applicable to QCD, and thereby provide the impetus and the foundation for the massive computational effort required to complete the task. The effort *is* massive but then so was the development of lattice gauge theory.

Other methods are also candidates for calculations in QCD. Among them are Dyson–Schwinger equations [8], which is also a Euclidean method; the truncated conformal space approach [9]; and the transverse lattice [10], which combines a lattice in transverse coordinates with two-dimensional light-front quantization for longitudinal space and time. These are quite adequately addressed elsewhere.

Perturbative calculations can also benefit from a light-front approach; however, these are also outside the scope of the present review. Instead, the recent review by Cruz-Santiago, Kotko, and Staśto [11] provides an excellent introduction to light-front calculations of scattering amplitudes.

The focus here is on light-front Hamiltonian methods for nonperturbative bound-state problems. The methods are, at least loosely, based on Fock-state expansions of the eigenstates. The Fock states are eigenstates of momentum, particle number, and fundamental quantum numbers associated with any symmetries or charges. The wave functions appear as the coefficients of the Fock states in the expansion; they are functions of (relative) momenta[1] and are indexed by the particle count and quantum numbers. The Hamiltonian eigenvalue problem is then transformed into a coupled set of integral equations for these wave functions, with the invariant mass of the eigenstate as the eigenvalue. As such, the approach lends itself well to numerical solution by discretization [12,2] and by basis-function expansions [13,14].

The light-front Fock vacuum is an eigenstate of the full Hamiltonian, including interactions, provided that zero modes are excluded [2]. The solution of the eigenvalue problem for the light-front Hamiltonian can then focus on the massive states, unlike equal-time quantization where the vacuum state itself must be computed as well. This is a significant advantage for light-front quantization. As is the added characteristic that vacuum contributions are absent from the Fock-state expansions of the massive states. The Fock-state wave functions can then be interpreted as defining the massive state itself.

However, this is a much weaker statement about the vacuum than to claim that the light-front Fock vacuum is the physical vacuum. The latter is not empty, and any physics that, in equal-time quantization flows from the structure of the physical vacuum is typically difficult to reproduce in light-front quantization. One exception to this is a light-front derivation of the Casimir effect [15], but quantities such as critical couplings and exponents in $\phi^4$ theory remain elusive.

To solve the infinite system of equations for the masses and wave functions requires some form of truncation. This is usually done as a truncation in Fock space to maximum numbers of particle types. However, such a truncation causes uncancelled divergences because cancellations between contributions to a particular process frequently require contributions from (disallowed) intermediate states with additional particles. Two solutions to this difficulty have been proposed. One is sector-dependent renormalization [16–19], where bare parameters of the Lagrangian are allowed to depend on the Fock sector or sectors on which the particular interaction term acts. The uncancelled divergences are absorbed

[1] Unlike equal-time coordinates, light-front coordinates admit a separation of external and relative momenta.