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Review

The many facets of the (non-relativistic) Nuclear Equation of State

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ABSTRACT

A nucleus is a quantum many body system made of strongly interacting Fermions, protons and neutrons (nucleons). This produces a rich Nuclear Equation of State whose knowledge is crucial to our understanding of the composition and evolution of celestial objects. The nuclear equation of state displays many different features; first neutrons and protons might be treated as identical particles or nucleons, but when the differences between protons and neutrons are spelled out, we can have completely different scenarios, just by changing slightly their interactions. At zero temperature and for neutron rich matter, a quantum liquid–gas phase transition at low densities or a quark–gluon plasma at high densities might occur. Furthermore, the large binding energy of the α particle, a Boson, might also open the possibility of studying a system made of a mixture of Bosons and Fermions, which adds to the open problems of the nuclear equation of state.

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1. Introduction

Many aspects of the Nuclear Equation of State (NEOS) have been studied in large detail in the past years. Finite nuclei resemble classical liquid drops, the crucial difference is that the nucleus in its ground state, or zero temperature, does not 'solidify' similarly to a drop at low temperatures [1–7]. This is due to the quantum nature of the nucleus: more specifically its constituents, neutrons (n) and protons (p), are Fermions. They obey the Pauli principle which forbids two equal Fermions, two protons with the same spin or two neutrons with the same spin (either both up or both down), to occupy the same quantum state. Thus at zero temperature, two or more Fermions cannot be at rest (a solid) when confined in a finite volume. In intuitive terms, we can express the Pauli principle by saying that a volume $V = \frac{4\pi}{3}R^3$ in coordinate space and $V_p = \frac{4\pi}{3}p_F^3$ in momentum space of size $h^3 = (2\pi\hbar)^3$ can at most contain $g = (2s+1)(2\tau+1)$ nucleons, where $\hbar = 197.3$ MeV fm is the Planck constant, s and τ are the spin and isospin of the considered Fermion. Thus:

$$\frac{\frac{4\pi}{3}R^3\frac{4\pi}{3}p_F^3}{h^3} = \frac{A}{g}. \tag{1}$$

Since the density is given by $\rho = \frac{A}{V}$, where $A = N + Z$ is the total number of nucleons (protons+neutrons), we can easily invert equation (1) and express the Fermi momentum p_F as a function of density [8,9]:

$$p_F = \left(\frac{3\rho}{4\pi g}\right)^{1/3} h. \tag{2}$$

For a nucleus in the ground state $\rho_0 = 0.16$ fm^{-3} and $p_F = 263$ MeV/c. This means that the nucleons in the nucleus are moving, even at zero temperature, with a maximum momentum p_F corresponding to a Fermi energy $\epsilon_F = \frac{p_F^2}{2m} = 36.8$ MeV. Because of the Fermi energy, the nucleus or any Fermionic system would expand if there is no confining external potential or interactions among them. Since the total energy of a nucleus in its ground state is about $E \approx -8$ A MeV, the average kinetic energy from Fermi motion is $\frac{3}{5}\epsilon_F = 22.08$ MeV/A, then the interaction must account for an average -30 MeV/A, which is a large value. Because of the relentless motion of the nucleons in the nuclei confined to a finite space due to the nuclear force, we can compare the nucleus to a drop or a liquid. Similarly to a drop, we can compress it and it will oscillate with a typical frequency known as Isoscalar Giant Monopole Resonance (ISGMR) [10–21]:

$$E_{GMR} = 80A^{-1/3} \approx \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}, \tag{3}$$

where $K_A = K +$ surface, Coulomb, symmetry and pairing corrections [21], $\langle r^2\rangle = \frac{3}{5}R^2$, $R = r_0A^{1/3} = 1.14A^{1/3}$ fm is the average radius of a nucleus of mass A, and K is the nuclear compressibility which could be derived from the NEOS if known. From experiments and comparison to theory we know that $K = 250 \pm 25$ MeV which implies that the nucleus is quite 'incompressible'. Other nuclear modes are possible where the volume of the nucleus remains constant, such as shape oscillations. A significant example of shape oscillation is the Isoscalar Giant Quadrupole Resonance (ISGQR) mode. Most of these oscillations can be described quantum mechanically as we will discuss later, but also, in some limit, using hydrodynamics [16,22,23]. An important and maybe crucial feature of nuclei is the fact that its constituents, protons and neutrons, can be described as two different quantum fluids. The fluids might behave as one fluid, such as in the Giant Resonance (GR) cases we briefly discussed before and therefore called Isoscalar GR (ISGR). There are resonances where n and p oscillate against each other and these are called Isovector GR (IVGR). An important example, related to this last feature, is the Isovector Giant Dipole Resonance (IVGDR). We will discuss this case in some detail below, Section 8, since it gives important information about the n–p restoring force or potential, thus giving important constraints about the NEOS.

The situations discussed above regard the nucleus near its ground state. However, important phenomena and objects in the universe, such as the Big-Bang (BB) [24–28], Supernovae explosions (SN) [28–31] or Neutron Stars (NS) [28,31–44] require the knowledge of the nuclear interactions in extreme situations, this means we need to pin down the NEOS not only near ρ_0 but also at very high or very low densities and/or temperatures. Because of the liquid drop analogy, we expect that if we decrease the density and increase the temperature, the system will become unstable and we will get a "quantum liquid–gas" (QLG) phase transition. Not only because the nucleus is a quantum system, but also because it is made of two strongly interacting fluids, thus the "symmetry energy", i.e. the energy of interaction between n and p, will be crucial. At very high densities, even at zero temperature, the nucleons will break into their constituents, quarks and gluons, and we get a state of matter called Quark–Gluon Plasma (QGP). Such a state occurred at the very beginning of the BB [24–28], at very high temperature, and it might occur in the center part of massive stars including neutron stars [28,31–44] as well as heavy ion collisions at relativistic energies [45–50]. It is very surprising that slightly changing the symmetry energy we can