Progress in Particle and Nuclear Physics 72 (2013) 44–98

Contents lists available at SciVerse ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

The renormalization scale-setting problem in QCD

Xing-Gang Wu [a,*], Stanley J. Brodsky [b], Matin Mojaza [b,c]

[a] Department of Physics, Chongqing University, Chongqing 401331, PR China
[b] SLAC National Accelerator Laboratory, Stanford University, CA 94039, USA
[c] CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230, Denmark

ARTICLE INFO

Keywords:
Renormalization group
Renormalization scale
BLM/PMC
QCD

ABSTRACT

A key problem in making precise perturbative QCD predictions is to set the proper renormalization scale of the running coupling. The conventional scale-setting procedure assigns an arbitrary range and an arbitrary systematic error to fixed-order pQCD predictions. In fact, this *ad hoc* procedure gives results which depend on the choice of the renormalization scheme, and it is in conflict with the standard scale-setting procedure used in QED. Predictions for physical results should be independent of the choice of the scheme or other theoretical conventions. We review current ideas and points of view on how to deal with the renormalization scale ambiguity and show how to obtain renormalization scheme- and scale-independent estimates. We begin by introducing the renormalization group (RG) equation and an extended version, which expresses the invariance of physical observables under both the renormalization scheme and scale-parameter transformations. The RG equation provides a convenient way for estimating the scheme- and scale-dependence of a physical process. We then discuss self-consistency requirements of the RG equations, such as reflexivity, symmetry, and transitivity, which must be satisfied by a scale-setting method. Four typical scale setting methods suggested in the literature, *i.e.,* the Fastest Apparent Convergence (FAC) criterion, the Principle of Minimum Sensitivity (PMS), the Brodsky–Lepage–Mackenzie method (BLM), and the Principle of Maximum Conformality (PMC), are introduced. Basic properties and their applications are discussed. We pay particular attention to the PMC, which satisfies all of the requirements of RG invariance. Using the PMC, all non-conformal terms associated with the β-function in the perturbative series are summed into the running coupling, and one obtains a unique, scale-fixed, scheme-independent prediction at any finite order. The PMC provides the principle underlying the BLM method, since it gives the general rule for extending BLM up to any perturbative order; in fact, they are equivalent to each other through the PMC–BLM correspondence principle. Thus, all the features previously observed in the BLM literature are also adaptable to the PMC. The PMC scales and the resulting finite-order PMC predictions are to high accuracy independent of the choice of the initial renormalization scale, and thus consistent with RG invariance. The PMC is also consistent with the renormalization scale-setting procedure for QED in the zero-color limit. The use of the PMC thus eliminates a serious systematic scale error in perturbative QCD predictions, greatly improving the precision of empirical tests of the Standard Model and their sensitivity to new physics.

Contents

* Corresponding author.
E-mail address: wuxg@cqu.edu.cn (X.-G. Wu).

0146-6410/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ppnp.2013.06.001

1. Introduction

Quantum chromodynamics (QCD) is believed to be the field theory of hadronic strong interactions. Due to its asymptotic freedom property [1,2], the QCD running coupling becomes numerically small at short distances, allowing perturbative calculations of cross sections for high momentum transfer physical processes. In the perturbative QCD (pQCD) framework, a physical quantity (ρ) is expanded to n-th order in the QCD coupling $\alpha_s(\mu_r)$; i.e.,

$$\rho_n = \mathcal{C}_0\, \alpha_s^p(\mu_r) + \sum_{i=1}^{n} \mathcal{C}_i(\mu_r)\, \alpha_s^{p+i}(\mu_r), \quad (p \geq 0) \tag{1}$$

where $\mathcal{C}_0$ is the tree-level term, $\mathcal{C}_1$ the one-loop correction, $\mathcal{C}_2$ the two-loop correction, etc., and p is the power of the coupling associated with the tree-level term. The renormalization scale μ_r must be specified in order to obtain a definite prediction. The calculation of the coefficients $\mathcal{C}_i(\mu_r)$ involves ultraviolet divergences which must be regulated and removed by a renormalization procedure. The infinite series $\rho_{n\to\infty}$ is in principle renormalization scheme and renormalization scale independent because of renormalization group (RG) invariance [3–8]; i.e., the physical predictions of a theory, calculated up to all orders, are formally independent of the choice of renormalization scale and renormalization scheme. However, at any finite order, the scale/scheme dependence from $\alpha_s(\mu_r)$ and $\mathcal{C}_i(\mu_r)$ do not exactly cancel, leading to renormalization-scheme and renormalization-scale ambiguities. Such ambiguities are well-known [9–21]. A guiding principle for resolving such problems is that physical results must be independent of theoretical conventions.

It should be recalled that there is no ambiguity in setting the renormalization scale in quantum electrodynamics (QED) at any finite order. Mass renormalization is straightforward in QED. Due to the Ward–Takahashi identity [22], the divergences in the vertex and fermion wavefunction corrections exactly cancel, and the remaining ultraviolet divergence Z_3 associated with the vacuum polarization insertions defines a natural scale for the running QED coupling $\alpha_{em}(q^2)$. For example, the renormalization scale for the electron–muon elastic scattering due to the one-photon exchange skeleton graph in the conventional Gell Mann–Low (GM-L) scheme [4] is simply equal to the momentum transfer squared $t = q^2$ carried by