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Review

Chiral perturbation theory: An effective field theory

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Abstract

The concept of an effective field theory (EFT), *i.e.* a theory which is valid only in a limited range of parameter space, is one which has become highly developed recently. After presenting familiar examples from the realms of classical and quantum mechanics together with condensed matter physics, we show that QCD has an intriguing parallel with superconductivity in that, by using quark–antiquark (mesonic) degrees of freedom and exploiting the symmetries of the theory, a useful and powerful theory – chiral perturbation theory – results, which offers a complete and rigorous description of low energy particle and nuclear physics.

Keywords: Chiral symmetry; Goldstone bosons; Effective field theory

1. Introduction

There exist many situations in physics wherein two quite separate scales – one heavy and one light – are involved. In such cases, provided that one is working at an energy–momentum small compared to the heavy scale one can write the interaction in terms of an "effective" Hamiltonian, which is written only in terms of the light degrees of freedom but which fully incorporates all (virtual) effects associated with the heavy scale [1]. Such an effective interaction is often able to isolate and more clearly represent the underlying physics of a given process than a rigorous treatment involving the entire system. We present simple examples below from the realms of quantum mechanics, condensed matter, physics, and QCD.

2. Examples

2.1. Classical mechanics

A simple example from classical mechanics involves the use of the effective gravitational potential

$$V_{\text{eff}} = mg(r - R_E)$$

in the vicinity of the earth's surface rather than the full Newtonian form

$$V = -\frac{GmM_E}{r}$$

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which is valid everywhere [2]. As long as the motion involves only distances small compared to the earth's radius – $|r - R_E| \ll R_E$ – the calculational simplicity of the first – mgh – form allows all of the basic physics of the gravitational interaction to be extracted without the complexity required to treat motion involving the tiny variation of the gravitational constant with height.

2.2. Quantum mechanics

A particularly nice example of effective field theory (EFT) methods is found in quantum mechanics, from the classic problem of understanding why the sky is blue [3]. The answer, of course, comes from Compton scattering of visible light from the air atoms which make up the earth's atmosphere. First, however, we outline the full analysis. Using the simple Hamiltonian [4]

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + e\phi \tag{1}$$

and lowest-order perturbation theory, one finds the Kramers–Heisenberg amplitude

$$\begin{aligned}\text{Amp} = &-\frac{\mathrm{e}^2}{m\sqrt{2\omega_i 2\omega_f}}\left[\hat{\epsilon}_i\cdot\hat{\epsilon}_f + \frac{1}{m}\sum_n\left(\frac{\hat{\epsilon}_f\cdot\langle 0|\vec{p}\mathrm{e}^{-\mathrm{i}\vec{q}_f\cdot\vec{r}}|n\rangle\langle n|\vec{p}\mathrm{e}^{\mathrm{i}\vec{q}_i\cdot\vec{r}}|0\rangle\cdot\hat{\epsilon}_i}{\omega_i + E_0 - E_n}\right.\right.\\ &\left.\left.+\ \frac{\hat{\epsilon}_i\cdot\langle 0|\vec{p}\mathrm{e}^{\mathrm{i}\vec{q}_i\cdot\vec{r}}|n\rangle\langle n|\vec{p}\mathrm{e}^{-\mathrm{i}\vec{q}_f\cdot\vec{r}}|0\rangle\cdot\hat{\epsilon}_f}{E_0 - \omega_f - E_n}\right)\right].\end{aligned} \tag{2}$$

When the energy of the photons is small compared to a typical excitation energy, the use of simple quantum mechanical identities allows one to simplify Eq. (2) to the form [5]

$$\text{Amp} \propto \omega^2 a_0^3$$

where $a_0 = \alpha/m$ is the Bohr radius. The corresponding cross section $\sigma \sim |\text{Amp}|^2 \propto \omega^4$ and explains why the sky is blue—blue light is scattered *much* more strongly than its red counterpart.

A corresponding EFT discussion is much simpler. An effective interaction which describes Compton scattering from an atomic system must satisfy certain basic strictures. It must be quadratic in the vector potential $\vec{A}$, must be a rotational scalar, be gauge invariant, and be symmetric under spatial inversion and time reversal. After a little thought it becomes clear that the lowest order such effective Hamiltonian must have the form

$$H_{\text{eff}} = -\frac{1}{2}4\pi\alpha_E\vec{E}^2 - \frac{1}{2}4\pi\beta_M\vec{H}^2 \tag{3}$$

where α_E, β_M are simple constants—the electric and magnetic polarizabilities. Since $\vec{E}, \vec{H} \sim \omega\hat{\epsilon}$ we easily find the desired form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto |\langle f|H_{\text{eff}}|i\rangle|^2 \propto \alpha_E^2\omega^4, \beta_M^2\omega^4. \tag{4}$$

Obviously then EFT has isolated the basic physics of Rayleigh scattering with a minimum of formalism. Of course, the overall scale is also of importance here and can be obtained from the feature that the dimensionality of the polarizabilities must be that of volume and for large wavelengths, the only volume in the problem is that of the atom, which is $4\pi a_0^3/3$.

2.3. Superconductivity

A third example, this time from condensed matter physics, is that of superconductivity. In this case electron–lattice interactions lead, in a complex fashion, to the effective BCS Hamiltonian, which involves an effective attractive interaction between electron pairs near the Fermi surface which are anticorrelated in spin and momentum [6]

$$H_{BCS} = \sum_{\vec{k},s}\psi^\dagger_{\vec{k},s}\left(\mathrm{i}\frac{\partial}{\partial t} - \frac{\vec{k}^2}{2m} - \mu\right)\psi_{\vec{k},s} - g\sum_{\vec{k}}\psi^\dagger_{\vec{k},\uparrow}\psi^\dagger_{-\vec{k},\downarrow}\psi_{-\vec{k},\downarrow}\psi_{\vec{k},\uparrow} \tag{5}$$