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Review

Hadron properties and Dyson–Schwinger equations

C.D. Roberts

Physics Division, Argonne National Laboratory, Argonne IL 60439, USA

Abstract

An overview of the theory and phenomenology of hadrons and QCD is provided from a Dyson–Schwinger equation viewpoint. Following a discussion of the definition and realization of light-quark confinement, the nonperturbative nature of the running mass in QCD and inferences from the gap equation relating to the radius of convergence for expansions of observables in the current-quark mass are described. Some exact results for pseudoscalar mesons are also highlighted, with details relating to the $U_A(1)$ problem, and calculated masses of the lightest $J = 0, 1$ states are discussed. Studies of nucleon properties are recapitulated upon and illustrated: through a comparison of the ln-weighted ratios of Pauli and Dirac form factors for the neutron and proton; and a perspective on the contribution of quark orbital angular momentum to the spin of a nucleon at rest. Comments on prospects for the future of the study of quarks in hadrons and nuclei round out the contribution.

Keywords: Bethe–Salpeter equations; Confinement; Dynamical chiral symmetry breaking; Dyson–Schwinger equations; Covariant Faddeev equation; Meson spectroscopy; Nucleon form factors; Hadron physics

1. Introduction

In trying to elucidate the role of quarks in hadrons and nuclei one steps immediately into the domain of relativistic quantum field theory where within the key phenomena can only be understood via nonperturbative methods. Two prime examples are: confinement, the empirical fact that quarks have not hitherto been detected in isolation; and dynamical chiral symmetry breaking, which is responsible, amongst many other things, for the large mass splitting between parity partners in the spectrum of light-quark hadrons, even though the relevant current-quark masses are small. Neither of these phenomena is apparent in QCD's Lagrangian and yet they play a dominant role in determining the observable characteristics of real-world QCD. The physics of hadrons is ruled by such *emergent phenomena*.

2. Confinement

In connection with confinement it is worth emphasizing at the outset that the potential between infinitely-heavy quarks measured in numerical simulations of quenched lattice-regularized QCD – the so-called static potential – is simply not relevant to the question of light-quark confinement. In fact, it is quite likely a basic feature of QCD that a quantum mechanical potential between light-quarks is impossible to speak of because particle creation and annihilation effects are essentially nonperturbative.

E-mail address: cdroberts@anl.gov.

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A perspective on confinement was laid out in Ref. [1]. Expressed simply, confinement can be related to the analytical properties of QCD's Schwinger functions, which are often loosely called Euclidean-space Green functions. For example, it can be read from the reconstruction theorem that the only Schwinger functions which can be associated with expectation values in the Hilbert space of observables; namely, the set of measurable expectation values, are those that satisfy the axiom of reflection positivity [2]. This is an extremely tight constraint. It can be shown to require as a necessary condition that the Fourier transform of the momentum-space Schwinger function is a positive-definite function of its arguments. However, that is not sufficient.

In relation to 2-point Schwinger functions, which are those connected with the propagators of elementary excitations in QCD, the axiom of reflection positivity is satisfied if, and only if, the Schwinger function possesses a Källén–Lehmann representation. This statement is most easily illustrated for a scalar field, in which case it means that one can write the scalar-field's 2-point function in the form[1]

$$\mathcal{S}(p^2) = \int_0^\infty \mathrm{d}\varsigma \, \frac{\rho(\varsigma)}{p^2+\varsigma^2}, \quad \rho(\varsigma) \geq 0 \;\; \forall \varsigma \geq 0. \tag{1}$$

The spectral density for a noninteracting scalar field of mass m is $\rho(\varsigma) = \delta(\varsigma - m)$. It is plain that no function $\mathcal{S}(p^2)$ which falls-off faster than $1/p^2$ at large spacelike momenta can be expressed in this way.

An appreciation of the importance of the axiom associated with reflection positivity has led to the formulation of a *confinement test* [3]. With a momentum-space Schwinger function, $\mathcal{S}(p)$, in hand, the first step is to calculate

$$\Delta(\tau) = \int \mathrm{d}^3x \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \, \mathrm{e}^{\mathrm{i}p_4\tau + \mathrm{i}\vec{p}\cdot\vec{x}} \, \mathcal{S}(p), \tag{2}$$

which gives the configuration-space Schwinger function in the rest frame.[2] One then examines the properties of $\Delta(\tau)$.

If an asymptotic state of mass m is associated with this Schwinger function, then

$$\Delta(\tau) \overset{\tau\to\infty}{=} \frac{1}{2m}\,\mathrm{e}^{-m\tau}; \tag{3}$$

i.e. the Schwinger function is positive definite and the mass of the asymptotic, propagating state is given by[3]

$$-\lim_{\tau\to\infty} \frac{\mathrm{d}}{\mathrm{d}\tau} \ln \Delta(\tau) = m. \tag{4}$$

If, on the other hand, $\Delta(\tau)$ calculated from a particular Schwinger function is not positive definite, then the axiom of reflection positivity is violated and the associated elementary excitation does not appear in the Hilbert space of observables. Thus the appearance of at least one zero in $\Delta(T)$ is a sufficient condition for confinement. It is a very clear signal.

An exemplar is provided by

$$\mathcal{S}(p) = \frac{p^2}{p^4 + 4\mu^4}, \tag{5}$$

a function for which with any particular choice of $\epsilon > 0$ there exists a $p_\epsilon^2 > 0$ such that $\forall p^2 > p_\epsilon^2$ one has $|\mathcal{S}(p) - 1/p^2| < \epsilon$. One calculates from Eq. (5):

$$\Delta(\tau) = \frac{1}{4\mu}\,\mathrm{e}^{-\mu\tau}\,[\cos\mu\tau - \sin\mu\tau]\,. \tag{6}$$

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1 A Euclidean metric will be used throughout. In concrete terms that means: for Dirac matrices, $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\gamma_\mu^\dagger = \gamma_\mu$; and $a \cdot b = \sum_{i=1}^4 a_i b_i$. A timelike vector, p_μ, has $p^2 < 0$. Naturally, no theory consistent with causality can produce a Schwinger function with a pole at spacelike p^2.

2 The arguments and conclusions that follow can readily be adapted to the case of models or theories without a mass gap; i.e. that possess a nonperturbatively massless excitation.

3 This picture is readily generalized to the case of a Schwinger function that describes a channel with more than one asymptotic state; i.e. a ground state plus excitations [4].