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Review

The QCD equation of state from the lattice

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ABSTRACT

The equation of state of QCD at finite temperatures and baryon densities has a wide range of applications in many fields of modern particle and nuclear physics. It is the main ingredient to describe the dynamics of experimental heavy ion collisions, the expansion of the early universe in the standard model era and the interior of compact stars. On most scales of interest, QCD is strongly coupled and not amenable to perturbative investigations. Over the past decade, first principles calculations using lattice QCD have reached maturity, in the sense that for particular discretisation schemes simulations at the physical point have become possible, finite temperature results near the continuum limit are available and systematic errors begin to be controlled. This review summarises the current theoretical and numerical state of the art based on staggered and Wilson fermions.

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1. Introduction

Quantum Chromodynamics (QCD) is the fundamental quantum field theory describing the strong interactions within the Standard Model. As such it is also the fundamental theory of nuclear matter. One of its many well-tested features is asymptotic freedom, according to which the coupling vanishes at asymptotically high energy scales revealing the nature of its constituents, the quarks and gluons. Conversely, the coupling is strong on hadronic energy scales $\lesssim 1$ GeV and we observe confinement of quarks and gluons which is not amenable to weak coupling expansions. Asymptotic freedom implies fascinating changes of dynamics when QCD is considered under extreme conditions, where either a high temperature sets the dominant momentum scale, as in the early universe, or a high baryon density, as in compact stars. Heavy ion collision experiments have succeeded in creating the hot quark gluon plasma in the laboratory and operate to further understand its properties, while future heavy ion experiments and astronomical observations aim to investigate cold and dense matter. Of particular phenomenological importance is the equation of state of the hot and/or dense system. As a fundamental property of the thermal system, it is in principle accessible to experiment allowing for a direct comparison with theoretical predictions. Moreover, the equation of state constitutes important input for further dynamical analyses of thermal systems, such as the hydrodynamical description of heavy ion collisions [1] or the search for new physics in the early universe [2].

Because of the inherently non-perturbative nature of the theory, numerical simulations of lattice QCD are the only tool allowing for predictions from first principles, for which it is known how to remove the associated systematic errors. Investigations of the equation of state have been going on for about two decades. After rapid initial successes with the pure gauge plasma, the step to include dynamical fermions has proved soberingly difficult. Systematic errors associated with fermion discretisations were initially underestimated, leading to apparent contradictions. However, after an impressive collective effort in man and machine power, these issues appear to be finally resolved. It was demonstrated that, while it may take some time and effort, systematic errors eventually *can* be controlled and removed, teaching us a lot about the underlying dynamics in the process. The equation of state at finite temperature and zero baryon density is now known for $N_f = 2 + 1$ quark flavours with physical masses very close to its continuum limit, which should be taken within the next year. This is based on the staggered fermion discretisation. Calculations with Wilson fermions are somewhat behind, but will soon serve as an independent cross check for remaining theoretical issues with the staggered formulation. Refinements to include the charm quark are also well on the way. The situation at finite density is much less satisfactory. Because of the sign problem of lattice QCD, direct simulations at finite baryon density are impossible and further approximations have to be made which are valid for sufficiently small chemical potentials. Nevertheless, important first steps in this direction have been made and we also understand the response of the equation of state to a small baryon chemical potential.