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Classical and quantum theory of the massive spin-two field



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ABSTRACT

In this paper, we review classical and quantum field theory of massive non-interacting spin-two fields. We derive the equations of motion and Fierz–Pauli constraints via three different methods: the eigenvalue equations for the Casimir invariants of the Poincaré group, a Lagrangian approach, and a covariant Hamilton formalism. We also present the conserved quantities, the solution of the equations of motion in terms of polarization tensors, and the tree-level propagator. We then discuss canonical quantization by postulating commutation relations for creation and annihilation operators. We express the energy, momentum, and spin operators in terms of the former. As an application, quark–antiquark currents for tensor mesons are presented. In particular, the current for tensor mesons with quantum numbers $J^{PC} = 2^{-+}$ is, to our knowledge, given here for the first time.

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1. Introduction

A first important step toward the understanding of elementary and composite particles in a relativistic context was made by O. Klein and W. Gordon: the so-called Klein–Gordon (KG) equation gives the correct relation between mass, energy, and momentum of all relativistic particles and is capable of describing the dynamics of scalar fields in the non-interacting limit. It can be used to study pions and other (pseudo-)scalar mesons as well as the recently discovered Higgs particle.

A fundamental property which naturally emerges when special relativity is applied to classical and quantum field theories is the spin, which is semi-integer for fermions and integer for bosons. P. Dirac introduced the famous Dirac equation for fermions with spin 1/2, which was able to describe relativistic electrons and leads to the correct energy levels of the hydrogen atom. The Dirac equation forms the basis for the description of all fundamental matter particles in the Standard Model, i.e., the quarks and leptons. It can also be used to describe composites of quarks, e.g. baryons with spin 1/2.

Later on, A. Proca [1] developed an equation which describes massive particles with spin one. Nowadays the Proca equation finds an application in effective theories for hadrons, in order to describe composite vector and axial-vector mesons, such as e.g. ρ and a_1 mesons. In the limit of zero masses, the Proca equation correctly reproduces the (inhomogeneous) Maxwell equation for the photon field.

At present, no fundamental particle with spin larger than one appears in the Standard Model. However, in an extension of the latter which encompasses the gravitational force, gravitons as spin-two particles might enter. On the other hand, composite particles with high spin exist: for instance,

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