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Energy flow along the medium-induced parton cascade

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ABSTRACT

We discuss the dynamics of parton cascades that develop in dense QCD matter, and contrast their properties with those of similar cascades of gluon radiation in vacuum. We argue that such cascades belong to two distinct classes that are characterized respectively by an increasing or a constant (or decreasing) branching rate along the cascade. In the former class, of which the BDMPS, medium-induced, cascade constitutes a typical example, it takes a finite time to transport a finite amount of energy to very soft quanta, while this time is essentially infinite in the latter case, to which the DGLAP cascade belongs. The medium induced cascade is accompanied by a constant flow of energy towards arbitrary soft modes, leading eventually to the accumulation of the initial energy of the leading particle at zero energy. It also exhibits scaling properties akin to wave turbulence. These properties do not show up in the cascade that develops in vacuum. There, the energy accumulates in the spectrum at smaller and smaller energy as the cascade develops, but the energy never flows all the way down to zero energy. Our analysis suggests that the way the energy is shared among the offsprings of a splitting gluon has little impact on the qualitative properties of the cascades, provided the kernel that governs the splittings is not too singular.

1. Introduction

The strongly collimated cascades of gluons that are radiated by an energetic parton is an essential ingredient of the physics of jets that are produced in various high energy collisions, including heavy ion

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collisions (for a recent review see [1] and references therein). This is because the inclusive distribution of the final particles is to a large extent determined by that of the radiated gluons at the time of hadronization, a feature known as the local parton–hadron duality [2]. In the case of heavy ion collisions, it is expected that multiple interactions of the hard partons with the deconfined matter present in the final state, induce another type of cascade [3–5]. Its elementary radiation process is described by the BDMPS-Z theory [6–11]. In the ideal case where only medium induced processes are retained, the resulting cascade exhibits particular features that make it very different from the cascade of gluons generated by radiation in vacuum, commonly described by the DGLAP evolution equation [12–14]. In particular, the BDMPS cascade exhibits scaling properties [3,5,22], akin to wave turbulence [15], that are apparently absent in the DGLAP cascade. Also, as was recently emphasized, the BDMPS cascade provides a natural mechanism for the transport of energy towards large angles, which contrasts with the strong angular ordering of QCD cascades in vacuum [16–19].

The main motivation of the present paper is to analyze the origin of the qualitative differences between the two types of cascades, with the goal of fostering our understanding of the BDMPS cascade. This cascade, as we have just recalled, may play an important role in determining the properties of jets produced in ultra relativistic heavy ion collisions. Insight will be obtained by comparing its properties with those of the more familiar DGLAP cascade. As we shall see, a quantity that plays a crucial role in this comparison is the characteristic time, $t_*(\omega)$, that it takes a gluon of energy ω to split into two other gluons. We refer to this time as the branching time. In the BDMPS cascade the branching time is an increasing function of ω, $t_*(\omega) \sim \sqrt{\omega}$, so that it decreases along the cascade as gluons of smaller and smaller energies are emitted. In other words, the branchings accelerate, and as a result it takes a finite time to transport a finite amount of energy from the leading particle to gluons carrying vanishingly small amounts of energy. This time, which we shall refer to as the stopping time [20], is infinite for the DGLAP cascade in which $t_*(\omega)$ is independent of ω.

Further features of the cascades follow from this crucial difference in the branching times. Consider for instance the energy distribution $D(\omega)$. In the BDMPS cascade this exhibits a persistent scaling behavior all the way down to $\omega = 0$, of the form $D(\omega) \sim 1/\sqrt{\omega}$, while in the DGLAP cascade scaling cannot be achieved in a finite time. The emergence of such a scaling relates to the existence of a stationary solution to the equation that governs the evolution of the energy distribution as a function of time. This stationary solution is of the form

$$D_{\rm st}(\omega) = \frac{t_*(\omega)}{\omega}, \tag{1.1}$$

and is associated to a constant (independent of ω) flow of energy,

$$\mathcal{F}(\omega) \equiv \frac{\partial \mathcal{E}(\omega)}{\partial t} \sim \frac{\omega D_{\rm st}(\omega)}{t_*(\omega)} = \text{const.} \tag{1.2}$$

where $\mathcal{E}(\omega)$ stands for the total energy carried by gluons with energies larger than ω, i.e., $\mathcal{E}(\omega) = \int_\omega {\rm d}\omega' D(\omega')$. The existence of a constant flow of energy at arbitrarily small ω implies that

$$\mathcal{E}(\omega_0) = \int_{\omega_0} {\rm d}\omega D_{\rm st}(\omega) = \int_{\omega_0} \frac{{\rm d}\omega}{\omega} t_*(\omega) \tag{1.3}$$

remains finite when ω_0 tends to zero. This forces $t_*(\omega)$ to be an increasing function of ω, which is indeed the case for the BDMPS cascade, for which $t_*(\omega) \sim \sqrt{\omega}$. In the DGLAP cascade $t_*(\omega)$ is constant, i.e. independent of ω, and the integral of the stationary scaling spectrum diverges. As we shall see, a scaling spectrum $D(\omega) \sim 1/\omega$ would also be expected in this case, but it takes an infinite time to develop.

The properties of the cascades depend also on the splitting kernel, that is, on the way the energy is distributed between the offsprings during a splitting. However, this turns out to have a minor effect on the main characteristics of the cascade, as compared to that of the transport time scale just mentioned, at least as long as the splitting kernel is not too singular. In that case the cascades develop as if the branching were completely democratic, with the two offsprings taking each half the energy of the parent gluon. The interactions responsible for the splittings can then be considered as local (in energy