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Quantum recurrence and fractional dynamic localization in ac-driven perfect state transfer Hamiltonians

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ABSTRACT

Quantum recurrence and dynamic localization are investigated in a class of ac-driven tight-binding Hamiltonians, the Krawtchouk quantum chain, which in the undriven case provides a paradigmatic Hamiltonian model that realizes perfect quantum state transfer and mirror inversion. The equivalence between the ac-driven single-particle Krawtchouk Hamiltonian $\hat{H}(t)$ and the non-interacting ac-driven bosonic junction Hamiltonian enables to determine in a closed form the quasi energy spectrum of $\hat{H}(t)$ and the conditions for exact wave packet reconstruction (dynamic localization). In particular, we show that quantum recurrence, which is predicted by the general quantum recurrence theorem, is *exact* for the Krawtchouk quantum chain in a dense range of the driving amplitude. Exact quantum recurrence provides perfect wave packet reconstruction at a frequency which is *fractional* than the driving frequency, a phenomenon that can be referred to as fractional dynamic localization.

1. Introduction

Quantum mechanical spreading of a particle hopping on a tight binding lattice is known to be suppressed by the application of an external force, which leads to periodic wave packet reconstruction [1,2]. The phenomenon of quantum diffusion suppression and periodic wave packet relocalization

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induced by an ac force was originally predicted in a seminal paper by Dunlap and Kenkre [2], and since then it is referred to as dynamic localization (DL). This kind of localization is conceptually very different from other forms of localization, like Anderson localization or localization in the periodically kicked quantum rotator problem, and arises from a collapse of the quasi energy spectrum at certain 'magic' driving amplitudes [3]. DL was observed in different physical systems, including electronic transport in semiconductor superlattices [4], cold atoms and Bose–Einstein condensates in optical lattices [5] and optical beams in curved waveguide arrays [6,7]. Several works have extended the theory of DL to different physical conditions, for example to account for disorder [8], non-nearest neighbor hopping [9], particle interaction [10], and to non-Hermitian lattices [11]. Lattice truncation is generally believed to be detrimental for perfect wave packet reconstruction. Indeed, quasi energy level collapse becomes only approximate for lattices with a finite number of sites [3,12,13]. In this case, even in the absence of the external ac field, approximate reconstruction should occur at long enough times according to the very general quantum recurrence theorem [14,15], which is an extension to a quantum system with a discrete energy (or quasi energy) spectrum of the celebrated Pioncaré recurrence theorem of classical mechanics [16].

In this work we present an exactly-solvable ac-driven lattice model with a finite number of lattice sites in which perfect wave packet reconstruction can occur at a frequency which is *fractional* than the driving frequency, a phenomenon that we refer to as *fractional* DL. As opposed to ordinary DL, fractional DL corresponds to a set of partial quasi energy collapses and can occur at almost every amplitude of the driving force. Fractional DL can be viewed as the realization of *exact* quantum recurrence in a system with a discrete quasi energy spectrum at finite times which are integer multiplies than the forcing oscillation period. The lattice model that we consider is the Krawtchouk quantum chain [17–19], which has been extensively investigated in several physical fields [17–21] for its property of realizing perfect quantum state transfer and mirror inversion in the undriven case [19,20].

The paper is organized as follows. In Section 2 we briefly introduce the ac-driven Krawtchouk quantum chain and show its equivalence with the ac-driven double-well model for non-interacting bosons (ac-driven bosonic junction). A closed-form expressions of the quantum propagator and quasi energy spectrum are derived in Section 3, where the phenomena of quantum recurrence, fractional DL and mirror inversion are discussed. Numerical results of quasi energy spectra, perfect wave packet reconstruction and mirror imaging arising from fractional DL are presented in Section 4. Finally, in Section 5 the main conclusions are outlined.

2. ac-driven Krawtchouk quantum chain and non-interacting bosonic junction equivalence

We consider the hopping motion of a quantum particle on a linear chain, composed by $(N+1)$ sites, driven by an external time-dependent force $F(t)$. In the nearest-neighbor tight-binding approximations, the quantum system is described by the time-dependent Hamiltonian

$$\hat{H}(t) = -\sum_{n=0}^{N} \kappa_n \left(|n+1\rangle \langle n| + |n\rangle \langle n+1| \right) + F(t) \sum_{n=0}^{N} n|n\rangle \langle n| \tag{1}$$

where $|n\rangle$ is the Wannier state that localizes the particle at lattice site n and κ_n is the hopping rate between adjacent sites n and $(n+1)$. By expanding the particle quantum state $|\psi(t)\rangle$ as a superposition of localized Wannier states $|\psi(t)\rangle = \sum_{n=0}^{N} c_n(t)|n\rangle$, the evolution equations for the amplitude probabilities c_n derived from the Hamiltonian (1) (with $\hbar = 1$) explicitly read

$$i\frac{dc_n}{dt} = -\kappa_n c_{n+1} - \kappa_{n-1} c_{n-1} + nF(t)c_n \tag{2}$$

$(n = 0, 1, 2, \ldots, N)$ with $\kappa_{-1} = \kappa_N = 0$. The Krawtchouk quantum chain corresponds to the following special sequence of hopping rates [17–20]

$$\kappa_n = \nu\sqrt{(n+1)(N-n)} \tag{3}$$

$(n = 0, 1, 2, \ldots, N)$, where ν is a constant parameter. In the undriven case, i.e. for $F(t) = 0$, the properties of the Krawtchouk quantum chain are well known and have been extensively investigated,