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Weak values and weak coupling maximizing the output of weak measurements

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HIGHLIGHTS

- We have provided a general framework to find the extremal values of a weak measurement.
- We have derived the location of the extremal values in terms of preparation and postselection.
- We have devised a maximization strategy going beyond the limit of the Schrödinger–Robertson relation.

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ABSTRACT

In a weak measurement, the average output $\langle o\rangle$ of a probe that measures an observable $\hat{A}$ of a quantum system undergoing both a preparation in a state ρ_{i} and a postselection in a state E_{f} is, to a good approximation, a function of the weak value $A_w = \mathrm{Tr}[E_{\mathrm{f}}\hat{A}\rho_{\mathrm{i}}]/\mathrm{Tr}[E_{\mathrm{f}}\rho_{\mathrm{i}}]$, a complex number. For a fixed coupling λ, when the overlap $\mathrm{Tr}[E_{\mathrm{f}}\rho_{\mathrm{i}}]$ is very small, A_w diverges, but $\langle o\rangle$ stays finite, often tending to zero for symmetry reasons. This paper answers the questions: what is the weak value that maximizes the output for a fixed coupling? What is the coupling that maximizes the output for a fixed weak value? We derive equations for the optimal values of A_w and λ, and provide the solutions. The results are independent of the dimensionality of the system, and they apply to a probe having a Hilbert space of arbitrary dimension. Using the Schrödinger–Robertson uncertainty relation, we demonstrate that, in an important case, the amplification $\langle o\rangle$ cannot exceed the initial uncertainty σ_o in the observable $\hat{o}$, we provide an upper limit for the more general case, and a strategy to obtain $\langle o\rangle \gg \sigma_o$.

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1. Introduction

In 1988 [1], Aharonov et al. introduced the concept of postselected weak measurements, initiating a prolific avenue of research. Recently, there have been several works considering the possibility of using weak and intermediate strength interaction in order to reconstruct an unknown quantum state [2–9] and to diminish the noise in a variable by preceding its measurement with the observation of the conjugate variable [10]. Other works, instead, in line with the initial proposal of Aharonov et al. [1], have focused on the amplification effect of weak measurement [11–15]. (However, the actual advantage over techniques based on strong measurement has been questioned [16–19].) Indeed, the provocative title of the original paper by Aharonov et al. was "How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100", meaning that the average output of a detector which, in a strong measurement, would give $p = \pm\lambda/2$ as outputs, can be amplified to $\langle p\rangle \sim 100\lambda$ if the measurement is weak and the system is suitably postselected. The weakness of the measurement means that initially the uncertainty over the pointer variable is much larger than λ, the distance between the peaks expected in the strong regime. This means that initially the pointer may not be in the zero position, but it could read, e.g., $p_0 = 99\lambda$ or $p_0 = -101\lambda$, etc. However, if the detector were a classical object, these fluctuations would cancel out on the average. Instead, as the detector obeys quantum mechanics as well, if it is prepared in a suitably quantum coherent superposition of pointer states [20,21] and if the system is postselected appropriately, this cancellation does not occur, possibly leading to a large output.

How large can the average output be? According to the simple formula of Ref. [1] there are no bounds to it, but as it turns out, the formula breaks down when the output is largish. This has prompted the need to provide a more reliable formula, working also in the regime where the measurement strength is weak but the overlap between the preparation and the postselection is small [22–25].

For a spin 1/2, it is possible to work out an exact solution for an instantaneous interaction [20,21] and more generally for a nondemolition interaction of finite duration [22]. This has allowed to study the maximization of the output based on the exact expression in Ref. [22], and then in Refs. [26–28] with varying degrees of generality. Kofman et al. [25] considered some particular cases of the maximization for a detector with an infinite dimensional Hilbert space performing a canonical von Neumann measurement, i.e. using a position variable $\hat{q}$ to couple with the system and its conjugate variable $\hat{p}$ as the readout for the measurement. Furthermore, Ref. [25] only considered pure preparation and postselection, so that there may be, in principle, higher maxima or lower minima for mixed preparation and postselection. During the completion of the present manuscript, a preprint appeared [29] that treats the problem of the maximization of the output as well. While the variational approach of Ref. [29] allows to include higher order corrections for a system of dimension $d > 2$, it applies only when q has a continuous spectrum and the readout p is its conjugate variable, and the procedure relies on a numerical approach. To the best of my knowledge, there is no systematic study of the maximization of an arbitrary output variable $\langle o\rangle$ for higher-dimensional systems. The main result of this paper is provided in Eqs. (25)–(27) and (30).

2. Background

2.1. Measurement model

As customary when treating weak measurements, it is supposed that a detector interacts with the measured system through the Hamiltonian $H = -\lambda\delta(t)\hat{q}\hat{A}$, i.e. the von Neumann model [30] of measurement is assumed. In this model, the output variable is usually taken to be $\hat{p}$, the conjugate variable of $\hat{q}$, i.e. $[\hat{q}, \hat{p}] = i$, with $[,]$ the commutator. Thus, $\hat{q}$ is assumed to have a continuous unbounded spectrum, so that it can be treated as a position operator. In the following, however, we shall not make this assumption, and in this sense we are diverging from the von Neumann model. Instead, we shall consider the output variable $\hat{o}$ to be arbitrary. Thus, the detector could have a finite-dimensional Hilbert space, for instance it could be a spin 1/2, with $\hat{q}$ a spin component, etc.