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Implication of Tsallis entropy in the Thomas–Fermi model for self-gravitating fermions

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HIGHLIGHTS

- Thomas–Fermi approach for self-gravitating fermions.
- A generalized Thomas–Fermi equation is derived.
- Nonextensivity preserves a scaling property of this equation.
- Nonextensive approach to Jeans' instability of self-gravitating fermions.
- It is found that nonextensivity makes the Fermionic system unstable at shorter scales.

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ABSTRACT

The Thomas–Fermi approach for self-gravitating fermions is revisited within the theoretical framework of the *q-statistics*. Starting from the *q-deformation* of the Fermi–Dirac distribution function, a generalized Thomas–Fermi equation is derived. It is shown that the *Tsallis entropy* preserves a scaling property of this equation. The *q-statistical* approach to Jeans' instability in a system of self-gravitating fermions is also addressed. The dependence of the Jeans' wavenumber (or the Jeans length) on the parameter q is traced. It is found that the *q-statistics* makes the Fermionic system unstable at scales shorter than the standard Jeans length.

1. Introduction

The Thomas–Fermi (TF) model [1,2] was originally introduced in order to describe the electron distribution and the Coulomb potential around the nucleus. It was used to describe in a semi-classical

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way, large atoms, solids as well as astrophysical objects. The model consists simply to consider the electrons as forming a gas of fermions, obeying the Fermi–Dirac distribution and occupying the phase space uniformly with two fermions per h^3, under the effect of the Pauli principle. The Poisson equation for the electrostatic potential gives the Thomas–Fermi equation, describing self-consistently the variation of the potential.

Surprisingly, a similar model was introduced to describe systems of fermions interacting via gravitational forces [3]. Such systems are ubiquitous in a variety of astrophysical situations, such as neutrons in a neutron star or neutrinos in models of dark matter. The Thomas–Fermi approach for such systems is based on the same hypothesis, except that the Poisson equation for the electrostatic field is now replaced by the Newton–Poisson equation (Newtonian limit of Einstein's equation) describing the gravitational field.

Though 85 years old, the TF model remains actual and is still subject to developments and new mathematical refinements [4–7]. Among them, a generalization in the recently introduced framework of Tsallis statistical mechanics [8,5]. The latter is a generalization of the conventional Boltzmann–Gibbs–Shannon (BGS) statistical mechanics, based on a one parameter generalization in the concept of the entropy in a way that it becomes non-additive *(see* [9] *and references therein for an actual view of the theory).* The deformed distributions arising from the Tsallis entropy have been recently observed in a variety of systems. Among these experimental evidences, the distribution characterizing the motion of cold atoms in dissipative optical lattices [10] or the velocity distributions in driven dissipative dusty plasma [11]. The *Tsallis statistical mechanics* seems to be the framework describing systems where the BGS statistics shows its limits: systems with long-range interactions, long time memory, systems evolving in a fractal space–time, or systems out of equilibrium. Systems of self-gravitating fermions may present such characteristics.

This paper is dedicated to a generalization of the TF model of self-gravitating fermions in the framework of *Tsallis statistical mechanics*. From the mathematical point of view, this procedure is identical to that already proposed by Martinenko and Shivamoggi in the atomic context [5]. However, one may expect that the effects *of the Tsallis q-statistics* have a more appreciable significance in the case of gravitational interactions. In fact, the Coulombian interactions have an effective short-range effect due to the Debye shielding, whereas the gravitational interactions are unshielded and have a long-range nature. Moreover, the thermodynamics of such systems shows some peculiar features, different from usual systems, such as a negative specific heat and an absence of global entropy maxima [12]. The introduction of *the q-statistics* for self-gravitating systems appears to be useful in many ways [13,14] and the fact that the gravity is a long-range force suggests that the self-gravitating system is one of the most preferable testing grounds for the framework of *Tsallis q-statistics* [12]. After a brief review of the TF model ideas and the *Tsallis statistics*, we will derive and discuss in the first section, the Thomas–Fermi equation describing a system of fermions interacting via Newtonian gravity. In the second section, we will explore the effect of *Tsallis entropy*on the Jeans instability of self-gravitating Fermionic matter. Concluding remarks are presented in the last section.

2. Generalized Thomas–Fermi equation

The *Tsallis statistical mechanics* is based on a one parameter generalization of the Shannon entropy. The generalized q-entropy reads [8]

$$S_q = k_B \frac{1 - \sum_i p_i^q}{q-1} \tag{1}$$

where p_i is the probability of the i-th microstate and q a real parameter measuring the degree of *non-additivity of the entropy and then the correlations in the system.* k_B stands for the Boltzmann constant and we will from now take it equal to unity. In the limit $q \to 1$, the generalized entropy reduces to the conventional one and its additivity is recovered. The Tsallis q-entropy leads to deformed distributions. For fermions, the latter reads

$$f^{(q)}(E) = \frac{1}{1 + [1 + \frac{(q-1)}{T}[E-\mu]]^{q/(q-1)}} \quad \text{with } 1 + \frac{(q-1)}{T}[E-\mu] > 0 \tag{2}$$