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Axial and transverse acoustic radiation forces on a fluid sphere placed arbitrarily in Bessel beam standing wave tweezers

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HIGHLIGHTS

- The axial and transverse forces on a fluid sphere in acoustical Bessel beams tweezers are evaluated.
- The attraction or repulsion to an equilibrium position in the standing wave field is examined.
- Potential applications are in particle manipulation using standing waves.

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ABSTRACT

The axial and transverse radiation forces on a fluid sphere placed arbitrarily in the acoustical field of Bessel beams of standing waves are evaluated. The three-dimensional components of the time-averaged force are expressed in terms of the beam-shape coefficients of the incident field and the scattering coefficients of the fluid sphere using a partial-wave expansion (PWE) method. Examples are chosen for which the standing wave field is composed of either a zero-order (non-vortex) Bessel beam, or a first-order Bessel vortex beam. It is shown here, that both transverse and axial forces can push or pull the fluid sphere to an equilibrium position depending on the chosen size parameter ka (where k is the wave-number and a the sphere's radius). The corresponding results are of particular importance in biophysical applications for the design of lab-on-chip devices operating with Bessel beams standing wave tweezers. Moreover, potential investigations in acoustic levitation and related applications in particle rotation in a vortex beam may benefit from the results of this study.

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1. Introduction

Particle entrapment using dual-beam acoustical tweezers [1] is the basis of a wealth of lab-on-a-chip applications, and constitutes the foundation for the development of (bio)acousto-fluidic devices [2–8]. The acoustical immobilization in fluids [9,10], gels [11,12] or curing epoxy [13–15], separation [16,17] and manipulation of particles [18–20] in dual-beam tweezers is achieved using counter-propagating waves that form a (quasi)-standing wave field. In the earlier applications, the acoustical tweezers used counter-propagating focused [1] or quasi-plane [21] waves, generated from finite acoustic sources.

More recently, the beam-forming design and tailoring of the incident waves in dual-beam acoustical tweezers is becoming an active topic of investigation [22–30]. Bessel beams [31–35], which overcome some of the limitations of plane waves or focused quasi-Gaussian waves [36,37] have been introduced for this purpose. Bessel beams represent "undistorted waves" [38] that propagate along an extended spatial distance without spreading defining a "limited diffraction" zone [39,40] when the beam is generated from a finite source. In some applications, it is desirable to lengthen this zone by using counter-propagating beams [41,42] for which a larger number of particles can be trapped over an extended region in space.

In dual-beam tweezers design, the interest has been particularly focused on evaluating the *axial* radiation force used to manipulate a spherical particle immersed in a non-viscous liquid, and centered on the axis of wave propagation of a circularly symmetric Bessel beam of standing waves [22–28]. Considering the more complex situation where the particle is placed arbitrarily (i.e. off-axially) in the acoustic field, only recently analytical models for predicting the acoustic radiation force have been developed based on a partial-wave expansion method [43,44] or an angular spectrum decomposition approach [45], and applied for Bessel beams of *progressive waves* [44]. There has also been some effort to apply ray acoustics methods to the analysis of the transverse acoustic radiation forces of focused beams [46]. However, the analysis based on ray acoustics does not account for the sphere's resonances that strongly influence the scattering [47–49] and radiation forces [44,50], and the ray acoustics approach is only valid for particles much smaller than the wavelength of the incident field. Other computational approaches based on lattice-Boltzmann simulations [51–53] and finite-difference time-domain (FDTD) [54] methods have been proposed, however, they are time and memory consuming for a full vectorial (3D) analysis of the acoustic radiation force and its related applications.

In this work, analytical closed-form series expansions for the axial and transverse radiation force components for Bessel standing waves are derived, and numerically evaluated of for a fluid (red blood) sphere immersed a nonviscous water solution, chosen as an example. The possibility of manipulating a fluid (red blood) sphere is an important application of particular interest in bioengineering and bio-acousto-microfluidics design, and an earlier [55] and recent [56] works demonstrated the successful levitation, trapping and measurement of red blood cells' compressibility in several different host environments using the acoustic radiation force of plane standing waves. The present analysis considers the case of a spherical fluid (red blood) cell considered perfectly spherical (to a first approximation) and placed arbitrarily in the field of *acoustical Bessel non-vortex and vortex beams composed of standing waves*. In the present analysis, the analytical expressions of the radiation force are given in terms of the beam-shape coefficients (BSCs) describing the incident standing wave-field and the scattering coefficients of the fluid sphere. Numerical examples for a zero-order (non-vortex) and a first-order Bessel (vortex) beam of standing waves of variable half-cone angle are considered, and calculations for the radiation force components reveal the existence of particle trapping behavior depending on the beam's parameters, the choice of the dimensionless frequency as well as the offset distance from the center of the beam. The BSCs are evaluated using a numerical integration procedure based on the discrete spherical harmonics transform [57,58], previously developed in the context of the generalized resonance scattering theory of arbitrary beams by a sphere/spherical shell in a non-viscous fluid. Several features of the results presented here may be related to corresponding experimental observations and potentially useful in experimental design of Bessel standing wave tweezers for biophysical applications. This study will assist in designing specific acoustical probes operating with Bessel beams for biological imaging [59], lab-on-a-chip and particle manipulation