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Exact theory of intermediate phases in two dimensions

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HIGHLIGHTS

- Phase separation with appearance of a third phase is studied exactly.
- Interfacial properties are derived from field theory.
- Exact solution of bulk wetting transition is provided.

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ABSTRACT

We show how field theory yields the exact description of intermediate phases in the scaling limit of two-dimensional statistical systems at a first order phase transition point. The ability of a third phase to form an intermediate wetting layer or only isolated bubbles is explicitly related to the spectrum of excitations of the field theory. The order parameter profiles are determined and interface properties such as passage probabilities and internal structure are deduced from them. The theory is illustrated through the application to the q-state Potts model and the Ashkin–Teller model. The latter is shown to provide the first exact solution of a bulk wetting transition.

1. Introduction

Statistical systems at a first order phase transition point allow for phase coexistence. Boundary conditions can be chosen to select a phase a on the left half of the system and a phase b on the right half, the two phases being separated by an interfacial region whose characterization is a particularly interesting problem. In systems allowing for a third degenerate phase, the latter can appear in the

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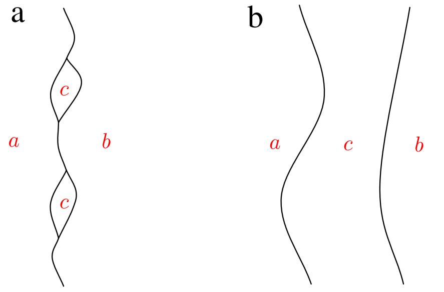

Fig. 1. Two different regimes of phase separation: a third phase appears in bubbles (a), or through a wetting layer (b).

interfacial region either via the formation of bubbles (or drops, Fig. 1a), or because a macroscopic ("wetting") intermediate layer of phase c forms between phases a and b (Fig. 1b). A transition from the first to the second regime induced by the variation of a parameter of the system goes under the name of wetting transition (see e.g. [1]).

The physics of phase separation is known to be sensitive to dimensionality. The two-dimensional case, in particular, possesses specific features originating from especially strong fluctuations of the interfaces. A key role in establishing the existence of these peculiarities was played by exact results for the planar Ising model [2], which then were used to test the reliability of heuristic descriptions (see in particular [3]). While the technical complexity of lattice derivations has restricted them to the Ising case, field theory should provide the natural framework for a general study of universal properties in the scaling limit. Nonetheless, a field theory of phase separation in two dimensions has been missing, arguably because the aforementioned peculiarities involve field theoretical counterparts. Only recently it has been shown [4,5] how the Ising results follow as particular cases of the general and exact field theoretical formalism which consistently takes into account the fact that interfaces in two dimensions correspond to trajectories of topological excitations (kinks) propagating in imaginary time.

In this paper we extend the formalism of [4] to study systems with a third phase in both regimes of Fig. 1 and the wetting transition. These systems are not of Ising type and have not been studied previously in a direct and exact way. Clearly, a main point is the characterization of the notion of interface. Being extended, interfaces are not fundamental objects of a local field theory. Hence we have to deduce their statistical properties from the determination of the spatial dependence of the order parameter, which is the expectation value of a local operator and the indicator of phase separation. We show how this analysis can be carried out in general in field theory and how it is intimately related to the connectedness properties of matrix elements on kink states.

We derive in particular the following properties. Whether the third phase is wetting or not is determined by the spectrum of kinks of the field theory. The interfacial tension between two phases coincides with the mass of the lightest kink connecting these two phases, and the equilibrium condition among the three interfacial tensions at the vertex of a bubble coincides with energy conservation for the relativistic particles at a bound state vertex. The transverse fluctuations of the interface in the non-wetting regime of Fig. 1a are Gaussian with a width increasing as $R^{1/2}$, where R is the size of the system in the direction parallel to the interface; the size in the transverse direction is assumed infinite, while R is taken much larger than the correlation length in the pure phases, which in turn is inversely proportional to the mass scale. The effect on the order parameter of the bubbles of Fig. 1a vanishes as $R^{-1/2}$ to leave a sharp separation between phases a and b in the asymptotic large R limit. For systems in which the external phases are exchanged by a symmetry, the coefficient of this bubble term depends only on the bulk theory and can also be determined exactly in many cases. The subsequent term in the large R expansion corresponds to trifurcations rather than bifurcations in Fig. 1a and is suppressed as R^{-1}; in two-phase, Ising-like systems this provides the first correction to sharp separation. In the wetting regime of Fig. 1b the order parameter profile does not approach at large R that corresponding to sharp separation between phases a and b. Its exact determination leads