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Coherent quantum states of a relativistic particle in an electromagnetic plane wave and a parallel magnetic field

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HIGHLIGHTS

- We study a relativistic electron in a particular electromagnetic field configuration.
- New exact solutions of the Klein–Gordon and Dirac equations are obtained.
- Coherent and displaced number states can describe a relativistic particle.

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ABSTRACT

We analyze the solutions of the Klein–Gordon and Dirac equations describing a charged particle in an electromagnetic plane wave combined with a magnetic field parallel to the direction of propagation of the wave. It is shown that the Klein–Gordon equation admits coherent states as solutions, while the corresponding solutions of the Dirac equation are superpositions of coherent and displaced-number states. Particular attention is paid to the resonant case in which the motion of the particle is unbounded.

1. Introduction

The relativistic description of a charged particle in an electromagnetic plane wave or a constant magnetic field is an old problem in quantum mechanics (see, e.g., [1–4]). Of particular interest is a combination of both fields: this problem was solved in its classical version by Roberts and Buchsbaum [5], and extended to quantum mechanics by Redmond [6], and Bergou and Ehlotzky [7], who

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showed that the solutions of the Klein–Gordon or Dirac equations for such a field configuration can be expressed in terms of number states of the harmonic oscillator.

Coherent states, which are superpositions of number states, were introduced in quantum optics by Glauber [8] and are widely used in quantum physics. Their generalization to fermions, however, is not straightforward. A consistent definition of fermionic coherent states in terms of Grassmann numbers was proposed by Cahill and Glauber [9]. Nevertheless, it remains to be seen if an equivalent description of spin 1/2 particles is possible at the level of first quantization.

As it will be shown in the present paper, the problem mentioned in the first paragraph provides a good example of how to describe a relativistic particle in terms of coherent quantum states. It presents no difficulty to define coherent states for a relativistic scalar particle, but for a particle described by the Dirac equation, it is necessary to resort, additionally, to the concept of a displaced number state as defined by de Oliveira et al. [10].

The plan of the present article is as follows: Section 2 provides a summary of the classical problem. Section 3 deals with the quantum version based on the Klein–Gordon equation; it is shown that this equation admits solutions in terms of the standard coherent states. The solution of the Dirac equation is presented in Section 4, and explicit expressions for the energy and current densities are given. In Section 5, it is shown that for particles of spin 1/2, the solution can be expressed in terms of coherent and displaced number states. Additionally, particular attention is paid to the special case of a resonance between the wave frequency and the Larmor frequency of the magnetic field; such a resonance produces an unbound motion of the particle.

2. Classical solution

Consider a particle of charge e and mass m in the field of a circularly polarized electromagnetic plane wave (a laser) with amplitude E and frequency ω, propagating along the z direction, and additionally a constant magnetic field, of magnitude B, in the same direction. These two fields are described, respectively, by the vector potentials

$$\mathbf{A}_L(t-z) = \frac{E}{\omega}\Big(\sin\omega(t-z), \cos\omega(t-z), 0\Big), \tag{1}$$

$$\mathbf{A}_B(\mathbf{r}) = \frac{B}{2}(-y, x, 0) \tag{2}$$

satisfying the Coulomb gauge. We set $c = 1 = \hbar$ in this article.

The classical problem of motion was solved by Roberts and Buchsbaum [5]. We summarize the main results in a form suited to our purpose. From the Lorentz force equation, it follows that $\mathcal{E}_0 \equiv \mathcal{E} - p_z$ is a conserved quantity, where $\mathcal{E}$ is the energy of the particle and $\mathbf{p}$ its momentum. Accordingly, the proper time of the particle is $\tau = k^{-1}(t-z)$, where $k = \mathcal{E}_0/m$. As for the components of the momentum, the equations of motion imply

$$\frac{d}{d\tau}\Sigma + i\omega_c\Sigma = -eEe^{-i\omega k\tau}, \tag{3}$$

where $\Sigma = p_x + ip_y$ and $\omega_c = eB/m$, and

$$\frac{d}{d\tau}p_z = \frac{eE}{m}(p_y\sin\omega k\tau - p_x\cos\omega k\tau). \tag{4}$$

These equations can be easily integrated. The solution for $\omega \neq \omega_c$ is

$$\Sigma(\tau) = \mathcal{C}e^{-i\omega_c\tau} + i\frac{eE}{\omega_c - k\omega}\Big(e^{-i\omega k\tau} - e^{-i\omega_c\tau}\Big) \tag{5}$$

where $\mathcal{C} \equiv \Sigma(0)$ is a complex constant of integration. This solution describes an oscillatory bound motion of the particle, the amplitude of oscillation being inversely proportional to $(k\omega - \omega_c)$.

In the resonant case $k\omega = \omega_c$, the above solution reduces to

$$\Sigma_{res}(\tau) = \Big(\mathcal{C} - eE\tau\Big)e^{-i\omega_c k\tau}. \tag{6}$$

The motion in this case is not bound: the particle describes a spiral trajectory.