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Dynamics of harmonically-confined systems: Some rigorous results

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HIGHLIGHTS

- We derive various rigorous results on the dynamics of harmonically-confined atomic gases.
- We derive an extension of the Harmonic Potential Theorem.
- We demonstrate the link between the energy absorption rate in a harmonically-confined system and the optical conductivity.

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ABSTRACT

In this paper we consider the dynamics of harmonically-confined atomic gases. We present various general results which are independent of particle statistics, interatomic interactions and dimensionality. Of particular interest is the response of the system to external perturbations which can be either static or dynamic in nature. We prove an extended Harmonic Potential Theorem which is useful in determining the damping of the centre of mass motion when the system is prepared initially in a highly nonequilibrium state. We also study the response of the gas to a dynamic external potential whose position is made to oscillate sinusoidally in a given direction. We show in this case that either the energy absorption rate or the centre of mass dynamics can serve as a probe of the optical conductivity of the system.

1. Introduction

The atomic gases in many cold atom experiments are confined in harmonic traps. An important consequence of this kind of confinement is that, in the absence of any additional external perturbation,

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the centre of mass of the system oscillates about the centre of the trap in simple harmonic motion without dissipation. This particular collective oscillation is referred to as the centre of mass or dipole mode. According to the generalized Kohn theorem [1,2], this behaviour is a generic property of a harmonically-confined system in which the interactions between particles depend only on their relative separation, and is independent of other intrinsic properties such as dimensionality, quantum statistics and the state of internal excitation. For these reasons, the undamped dipole oscillation can in fact be used to accurately determine the trapping frequencies [3] in situations where the experimental parameters defining the trapping potential are not known precisely. An additional, but more subtle, implication of such confinement is the content of the so-called Harmonic Potential Theorem (HPT) [4]. In essence, the HPT demonstrates the existence of a class of dynamical many-body states for which the probability density moves without change in shape. This theorem imposes important constraints on the form of approximate theories which deal with the dynamics of harmonically-confined many-body systems [4–6].

When the harmonicity of the confining potential is compromised, however, the centre of mass is coupled to the internal degrees of freedom and its dynamics becomes sensitive to the intrinsic properties of the system, including the specific form of the particle interactions. For this reason, the dipole oscillation can serve as an experimental diagnostic of various perturbations acting on the system. For instance, several experiments [7,9,8] have used dipole oscillations to study the transport of a Bose-condensate through a disordered medium or past a localized impurity. Although the motion of the condensate in these experiments does not lose its collectivity, dissipation does occur and leads to the damping of the centre of mass motion. Another experimental example is provided by the dipole oscillation of a trapped Bose gas in the presence of an optical lattice potential [10–13]. Here it was found that the dimensionality of the Bose gas plays a critical role in determining the way in which the centre of mass behaves as a function of time.

In all of these experiments, the dipole oscillation of the atomic system is initiated by an abrupt displacement of the trapping potential along a certain axial direction. If the displacement is large, the system begins its evolution in a highly non-equilibrium initial state. It is partly for this reason that much of the theoretical work dealing with the collective dynamics of Bose-condensed systems relies on numerical simulations of the time-dependent Gross–Pitaevskii (GP) equation [14–16]. One of the goals of this paper is to show that this nonequilibrium dynamics in the presence of the external perturbation can be considered from a different point of view when the system is harmonically confined. By means of an appropriate transformation, one can equivalently think of the system as being driven out of an initial *equilibrium* state by a *dynamic* external perturbation oscillating sinusoidally at the frequency of the trap. The availability of this alternate point of view is a consequence of what we refer to as the *extended* HPT. Its advantage is that the external perturbation can be treated by conventional linear response theory, at least when the perturbation is sufficiently weak. This approach was used effectively in an earlier paper [17] to determine the damping of the centre of mass motion due to a disorder potential.

A second purpose of this paper is to study the response of a harmonically-confined system to an external potential which is made to oscillate at an *arbitrary* frequency. Our discussion is motivated by a recent proposal [18] to probe the optical conductivity of a cold atomic gas in an optical lattice by shaking the lattice periodically along a certain direction. This is an interesting idea since it provides a method of addressing experimentally the optical conductivity of a system consisting of *neutral* atoms. However, the authors of Ref. [18] only considered bosons within a *uniform* lattice, while in most experiments the atoms are also subjected to a harmonic potential. In this paper, we show that one can also probe directly the optical conductivity of a gas that experiences a *combination* of a harmonic trapping potential and an arbitrary external potential when the latter is made to oscillate sinusoidally with a small amplitude. This generalization provides a precise link between theoretical calculations of the optical conductivity and possible experimental measurements on harmonically-confined gases in the presence of various external perturbations.

The rest of the paper is organized as follows. In Section 2, we provide a basic discussion of the dipole modes of a harmonically-confined system. In Section 3, we consider the response of a harmonically-confined system to a time-dependent homogeneous force. An explicit expression for the evolution operator of the system is obtained which motivates the introduction of a rather useful unitary displacement operator. These results are then used to provide an alternative derivation of the HPT. We