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Effective dynamics of a classical point charge

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HIGHLIGHTS

- Extension of the classical action principle for dissipative systems.
- New derivation of the Abraham–Lorentz force for a point charge.
- Absence of a runaway solution of the Abraham–Lorentz force.
- Acausality in classical electrodynamics.
- Renormalization of classical electrodynamics of point charges.

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ABSTRACT

The effective Lagrangian of a point charge is derived by eliminating the electromagnetic field within the framework of the classical closed time path formalism. The short distance singularity of the electromagnetic field is regulated by an UV cutoff. The Abraham–Lorentz force is recovered and its similarity to quantum anomalies is underlined. The full cutoff-dependent linearized equation of motion is obtained, no runaway trajectories are found but the effective dynamics shows acausality if the cutoff is beyond the classical charge radius. The strength of the radiation reaction force displays a pole in its cutoff-dependence in a manner reminiscent of the Landau-pole of perturbative QED. Similarity between the dynamical breakdown of the time reversal invariance and dynamical symmetry breaking is pointed out.

1. Introduction

Classical electrodynamics of point charges has an intrinsic length scale, the classical charge radius $r_0 = e^2/mc^2$, separating the well known classical domain from an unusual classical world, hidden behind quantum effects. Nevertheless it is an intriguing question how this classical world of point

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charges would look like in the absence of quantum mechanics. The attempts to uncover the dynamics around or beyond to the crossover scale, r_0, are rendered difficult by the current state of understanding of the radiation reaction force [1], an important ingredient of the electromagnetic interaction at these scales. The problem comes from the singularity of the electromagnetic field (EMF) in the point-like limit of the charges and appears as an instability, generated by the Abraham–Lorentz force, the runaway solution [2].

A simple, systematic derivation of the radiation reaction is presented here by working out the effective dynamics of a single point charge. The singularities of the point charge limit are regulated by smearing the interactions within an invariant length ℓ, an UV cutoff, and the dissipative nature of the radiation reaction force is handled by an extension of the variational method of classical mechanics, by recasting Schwinger's Closed Time Path (CTP) formalism [3]. Though this scheme has already been applied in a number of problems, its advantages and power have not yet been exhausted. It has been used to find relaxation in many-body quantum systems [4], to generate perturbation expansion for retarded Green functions in quantum field theory [5], to find a manifestly time reversal invariant description of quantum mechanics [6], to describe finite temperature effects in quantum field theory [7], to find the mixed state contributions to the density matrix by the path integral [8], to describe non-equilibrium processes [9], to derive equations of motion for the expectation value of local operators [10,11], to describe scattering processes with non-equilibrium final states [12] and finally, to derive radiation reaction force in QED [13–15] and effective quantum gravity [16]. We use it in this paper without referring to quantum theory, by generalizing the variational principle of classical mechanics [17–19]. This generalization provides a compromise which on the one hand, preserves the functional formalism of the variational principle, a natural way to introduce Green functions and perturbation expansion, and on the other hand, it allows the imposition of initial conditions and can handle dissipative forces.

The well known expression of the Abraham–Lorentz force is recovered in the calculation, together with the usual, UV divergent mass renormalization in the limit $\ell \to 0$. The linearized nonlocal equation of motion is derived for a point charge by retaining the full cutoff dependence. It is pointed out that we do not possess all data necessary to solve an initial condition problem and the solution is constructed in terms of the retarded Green function. The mode which drives the usual runaway solution is absent and the motion is causal for $\ell \gg r_0$. For sufficiently small values of ℓ unstable modes appear but they do not lead to runaway solutions because the retarded Green function relies on the unstable modes in the acausal regime where these modes are bounded. The elimination of the EMF generates UV singularities, in a manner similar to quantum field theories. This requires the introduction of an UV cutoff, a shift of the radiation field modes off the light-cone. It is pointed out that the integral over the world-line of the charges, contributing to the effective action is not uniformly convergent in the limit when the cutoff is removed, in a manner reminiscent of anomalies in quantum field theory. This similarity is made more explicit by showing that the EMF remains slightly off light-cone even after the removal of the cutoff.

The organization of this paper is the following. Section 2 introduces the CTP formalism in classical field theory which gives a systematic definition of the retarded Green function in classical effective theories. The way the time arrow is generated by the environment, irreversibility, acausality arise and the runaway solution is avoided are the subject of Section 3. Section 4 contains the derivation of the effective equation of motion. Finally, the summary of the results is presented in Section 6. Some details about the CTP Green function are collected in the Appendix.

2. CTP

An extension of the variational method of classical mechanics is needed to cover dissipative forces in a functional framework where the solution of initial condition problems can be found by means of a systematically derived retarded Green function. We consider a classical system described by the coordinate x, governed by the Lagrangian $L(x, \dot{x})$. A solution of the equation of motion for $t_i \le t \le t_f$ is usually identified by imposing auxiliary conditions, such as $x(t_i) = x_i$ and $x(t_f) = x_f$. Can we replace these boundary conditions with the initial conditions $x(t_i) = x_i,\ \dot{x}(t_i) = v_i$? The problem is that the equation of motion must then be imposed at the final time and it cancels the generalized