Annals of Physics 342 (2014) 270–282

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Searching for low-lying multi-particle thresholds in lattice spectroscopy

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HIGHLIGHTS

- Correlation-matrix projected correlators reveal more than one state contributing.
- Results are associated with strong mixing of single and multi-particle states in QCD.
- A two-exponential fit confirms the presence of two QCD eigenstates.
- The lower-lying eigenstate is consistent with a nucleon–pion scattering threshold.
- The impact of this small contamination on the higher-lying state is examined.

ARTICLE INFO

Article history:
Received 27 October 2013
Accepted 7 January 2014
Available online 15 January 2014

Keywords:
Lattice QCD
Odd-parity state
Pion–nucleon interaction
Scattering state
Multi-particle threshold

ABSTRACT

We explore the Euclidean-time tails of odd-parity nucleon correlation functions in a search for the S-wave pion–nucleon scattering-state threshold contribution. The analysis is performed using $2+1$ flavor $32^3 \times 64$ PACS-CS gauge configurations available via the ILDG. Correlation matrices composed with various levels of fermion source/sink smearing are used to project low-lying states. The consideration of 25,600 fermion propagators reveals the presence of more than one state in what would normally be regarded as an eigenstate-projected correlation function. This observation is in accord with the scenario where the eigenstates contain a strong mixing of single and multi-particle states but only the single particle component has a strong coupling to the interpolating field. Employing a two-exponential fit to the eigenvector-projected correlation function, we are able to confirm the presence of two eigenstates. The lower-lying eigenstate is consistent with a $N\pi$

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http://dx.doi.org/10.1016/j.aop.2014.01.004

scattering threshold and has a relatively small coupling to the three-quark interpolating field. We discuss the impact of this small scattering-state contamination in the eigenvector projected correlation function on previous results presented in the literature.

1. Introduction

The hadron spectrum provides an interesting foundational platform with which to investigate the QCD interactions of quarks and gluons. It presents significant challenges to current investigations of this relativistic quantum field theory. How do the resonances observed in experiment emerge from the first principles of QCD? What is the structure of these states and can it be linked to known effective degrees of freedom? For example, are elusive states like the $\Lambda(1405)$ or the nucleon Roper resonance exotic, perhaps having a molecular meson–baryon structure?

In this paper we address the first question by performing a lattice QCD study of the nucleon spectrum in a search for the multi-particle scattering threshold states which ultimately generate the finite width of the resonances in the infinite volume limit. Correlation matrices composed of traditional three-quark operators have been very successful in revealing a dense spectrum of baryon excited states in lattice QCD [1–12]. However the lowest lying multi-particle scattering state thresholds are often absent in the observed spectra.

The coupling of these two-particle dominated states to localized three-quark operators is suppressed relative to single-particle dominated states. In full QCD, 3-quark operators will have some coupling to the meson–baryon components of QCD eigenstates through interactions with the sea-quark loops of the QCD vacuum. However, this coupling is small relative to the coupling to the single-particle three-quark component of the eigenstate.

When the three-quark operator creates a resonance in the infinite volume limit, the overlap with a state dominated by a meson–baryon component is suppressed on the finite lattice volume, V, as $V^{-1/2}$. On large volumes these multi-particle dominated states will be difficult to observe with three-quark operators alone.

In the large-volume case, it is the mixing of one-, two- and multi-particle components in the finite-volume QCD eigenstates that predominantly governs the presence of multi-particle states when using traditional three-quark operators alone. As discussed in detail in the following, these multi-particle threshold scattering states are likely hidden within the projected correlation functions of correlation matrices composed purely of three-quark interpolating fields. Our focus here is to reveal these low-lying hidden states.

In the following, we report a case where two states are indeed participating in what otherwise would be considered to be an eigenstate-projected correlation function. Through a two-state analysis of the projected correlator we are able to accommodate this weakly coupled second state and evaluate the extent to which it influences the determination of the mass of the dominant state.

2. Correlation matrix techniques

To isolate energy eigenstates we use the correlation matrix or variational method [13,14]. To access N states of the spectrum, one requires a minimum of N interpolators. With the assumption that only N states contribute significantly to the correlation matrix G_{ij} at time t, the parity-projected two-point correlation function matrix for $\vec{p}=0$ can be written as

$$G^{\pm}_{ij}(t) = \sum_{\vec{x}} \mathrm{Tr}_{\mathrm{sp}}\,\{\Gamma_{\pm}\,\langle\, \Omega \mid \chi_i(x)\,\bar{\chi}_j(0) \mid \Omega\,\rangle\}, \quad (1)$$

$$= \sum_{\alpha}^{N} \lambda_i^{\alpha}\,\bar{\lambda}_j^{\alpha}\,e^{-m_\alpha t}, \quad (2)$$