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Casimir effect for a scalar field via Krein quantization

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HIGHLIGHTS

- A modification of QFT is considered to address the vacuum energy divergence problem.
- Casimir energy of a spherical shell is calculated, through this approach.
- In this technique, it is shown, the theory is automatically regularized.

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ABSTRACT

In this work, we present a rather simple method to study the Casimir effect on a spherical shell for a massless scalar field with Dirichlet boundary condition by applying the indefinite metric field (Krein) quantization technique. In this technique, the field operators are constructed from both negative and positive norm states. Having understood that negative norm states are un-physical, they are only used as a mathematical tool for renormalizing the theory and then one can get rid of them by imposing some proper physical conditions.

1. Introduction

The study of states with negative norm has a long story dating back to Dirac's work in 1942 [1]. Followed by Gupta and Bleuler in 1950, such states were used to remove the infrared (IR) divergence of QED [2]. In this way it is proved that quantization of the minimally coupled massless scalar field in de Sitter (dS) space can be done covariantly by the help of negative norm states [3,4]. In other words, due to the famous 'zero-mode' problem [3,5], one cannot define a proper dS invariant vacuum state

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with only positive norm states, so a Gupta–Bleuler type construction based on Krein space structure is needed [3]. This method provides a proposal to calculate graviton propagator in dS background in the linear approximation, without any pathological behavior for largely separated points [6]. (This result is in agreement with other works as long as it was shown that IR divergence of graviton propagator in one loop approximation do not appear in an effective way as a physical quantity because of gauge dependency [7–9].)

Utilizing Krein space method, in the calculation of the expectation value of energy–momentum tensor, the infinite term does not appear, it means that the vacuum energy vanishes without any need of reordering the operators. Furthermore, in the interacting QFT this method works well in removing the singular behaviors of Green's function at short relative distances (UV divergence) except the light cone singularity [10]. Recently, it was shown that the one-loop effective action for QED is regularized in a simple way in Krein space method [11]. The magnetic anomaly and Lamb shift are also studied in [12].

In Ref. [13], through this approach, the Casimir force between the parallel plates in flat space has been calculated which simply gave the correct result. In this paper, we study this effect for a spherical shell with the Dirichlet boundary condition in which the oscillation modes are defined in Krein Space. It is shown that this method has some advantages over the usual methods in discussion of QFT.

The layout of the paper is as follows: The method of Krein space is briefly reviewed in Section 2. In Section 3, through this method, we calculate the Casimir force and as it is shown, negative norm states automatically regularize the theory and after renormalization the results are the same as the previous related works. A brief discussion is done as a conclusion in Section 4. Finally, we have enclosed the paper with some details of mathematical calculations in Appendices A and B.

2. Krein quantization: basic set-up

Massless minimally coupled scalar field in dS space plays an important role in the inflationary models and the linear quantum gravity in dS space. Allen proved [5] that the covariant quantization of such a field in dS space cannot be constructed with only positive norm states. In Ref. [3], a new version of indefinite metric field quantization or Krein space quantization method was used in order to quantize the massless minimally coupled scalar field in dS space covariantly, in which one should consider both negative and positive frequency solutions to preserve the causality and also eliminate the IR divergences.

Let us illustrate Krein quantization by giving a simple example. A free scalar field $\phi(\vec{x}, t)$ which satisfies the Klein–Gordon equation

$$(\Box + m^2)\phi(x) = \left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\phi(\vec{x}, t) = 0, \tag{1}$$

has two sets of solutions:

$$u_P(\vec{k}, \vec{x}, t) = \frac{e^{i\vec{k}\cdot\vec{x} - iwt}}{\sqrt{(2\pi)^3 2w}}, \quad \text{and} \quad u_N(\vec{k}, \vec{x}, t) = \frac{e^{-i\vec{k}\cdot\vec{x} + iwt}}{\sqrt{(2\pi)^3 2w}}, \tag{2}$$

here the subscripts P and N are respectively referred to positive and negative frequency solutions and $w(\vec{k}) = k^0 = (\vec{k}\cdot\vec{k} + m^2)^{\frac{1}{2}} \geq 0$. The inner product is defined by

$$(\phi_1, \phi_2) = i\int_{t=\text{const}} d^3x(\phi_1^*\partial_t\phi_2 - \phi_2\partial_t\phi_1^*). \tag{3}$$

These modes are normalized by the following relations

$$\begin{aligned}
&(u_P(\vec{k}, \vec{x}, t), u_P(\vec{k}', \vec{x}, t)) = \delta(\vec{k} - \vec{k}'),\\
&(u_N(\vec{k}, \vec{x}, t), u_N(\vec{k}', \vec{x}, t)) = -\delta(\vec{k} - \vec{k}'),\\
&(u_P(\vec{k}, \vec{x}, t), u_N(\vec{k}', \vec{x}, t)) = 0.
\end{aligned} \tag{4}$$