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On the limits of quantum theory: Contextuality and the quantum–classical cut

George F.R. Ellis

Mathematics Department, University of Cape Town, South Africa

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ABSTRACT

This paper is based on four assumptions: 1. Physical reality is made of linearly behaving components combined in non-linear ways. 2. Higher level behaviour emerges from this lower level structure. 3. The way the lower level elements behaves depends on the context in which they are embedded. 4. Quantum theory applies to the lower level entities. An implication is that higher level effective laws, based on the outcomes of non-linear combinations of lower level linear interactions, will generically not be unitary; hence the applicability of quantum theory at higher levels is strictly limited. This leads to the view that both state vector preparation and the quantum measurement process are crucially based on top-down causal effects, and helps provide criteria for the Heisenberg cut that challenge some views on Schrödinger's cat.

1. Quantum theory and classicality

The classical to quantum relation is a key issue in understanding how quantum theory applies to the real world. In order to make progress in understanding this relation, it may well be profitable to consider first the way complexity emerges from the underlying physical relations, and second the way the operation of underlying physical processes is contextually determined. This paper will make the case that examining these issues of emergence and contextuality helps clarify the nature of the classical–quantum cut, also known as *Heisenberg's cut* ([1]:15), and hence the way that non-quantum macro behaviour can emerge from underlying quantum systems.

The basic viewpoint taken here is that physical theory must explain not only what happens in carefully controlled laboratory experiments, but also the commonplace features of life around us, for which we have a huge amount of evidence in our daily lives. We will set out this viewpoint in more detail in Section 2 below, after first setting out the fundamental quantum dilemma. Further

E-mail address: george.ellis@uct.ac.za.

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sections will explore the ways that quantum behaviour might emerge at higher levels of the hierarchy of complexity, will suggest contexts where this will almost certainly not be possible, and will explore the way contextual effects may help throw light on the quantum measurement problem.

This paper is structured as follows. Section 2 lays the foundations for the rest of the paper, setting out the context for the discussion and presenting a basic viewpoint which is then developed in the following sections. A key aspect is the proposal that higher level effective dynamics emerges out of lower level dynamics. Section 3 considers linear and non-linear aspects of quantum theory, leading to some criteria for when quantum physics will be valid, based on the essential linear aspects of the theory. Section 4 considers when the requisite linearity can emerge at higher levels in the hierarchy of complexity from lower level linear theories, and when it cannot emerge. Section 5 looks at the converse feature of how contextual effects from higher levels may influence lower level dynamics, giving a number of examples of top-down causation in the context of quantum physics. Section 6 looks at the issue of state vector reduction in the context of top-down causation from the local physical environment. Section 7 looks at implications of the discussion for the classical–quantum cut and Schrödinger's cat. Section 8 reviews the viewpoint presented, and considers issues that arise from the discussion as suitable subjects for further investigation.

A major issue that arises out of the discussion in Section 5 of top-down influences in physics is the origin of the arrow of time. This is discussed in a companion paper [2].

2. Foundations

This section sets out the basic foundations for the rest of the paper. Section 2.1 sets out the basics of quantum dynamics, Section 2.2 the elements of the measurement problem, and Section 2.3 sets out a basic standpoint that underlies what follows. Section 2.4 sets out the context of the hierarchy of structure, and Section 2.5 the viewpoint that both bottom-up and top-down causation take place in this hierarchy.

2.1. Basic dynamics

The basic expansion postulate of quantum mechanics [3–6] is that before a measurement is made, the state vector $|\psi\rangle$ can be written as a linear combination of unit orthogonal basis vectors

$$|\psi_1\rangle = \sum_n c_n |u_n(x)\rangle, \tag{1}$$

where u_n is an eigenstate of some observable $\hat{A}$ ([5]:5–7). The evolution of the system can be completely described by a unitary operator $\widehat{U}(t_2, t_1)$, and so evolves as

$$|\psi_2\rangle = \widehat{U}(t_2, t_1)\, |\psi_1\rangle. \tag{2}$$

Here $\widehat{U}(t_2, t_1)$ is the standard evolution operator, determined by the evolution equation

$$i\hbar \frac{d}{dt}|\psi_t\rangle = \hat{H}|\psi_t\rangle. \tag{3}$$

When the Hamiltonian $\hat{H}$ is time independent, $\widehat{U}$ has the form ([5]:102–103)

$$\widehat{U}(t_2, t_1) = e^{-\frac{i}{\hbar}\hat{H}(t_2 - t_1)} \tag{4}$$

which is unitary ([5]:109–113):

$$\widehat{U}\,\widehat{U}^\dagger = 1. \tag{5}$$

Applying this to (1) with $\widehat{U}(t_2, t_1)|u_n(x)\rangle = |u_n(x)\rangle$ (an invariant basis) gives

$$|\psi_2\rangle = \sum_n C_n |u_n(x)\rangle, \qquad C_n := \widehat{U}(t_2, t_1) c_n. \tag{6}$$